Recent progress in fast methods for the solution of Maxwell's equations

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Overview

- Introduction
- Boundary element techniques – method of moments

- Introduction to the MultiLevel Fast Multipole Algorithm (MLFMA)

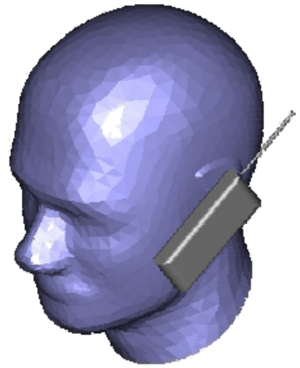
- Calderón preconditioning*
- Broadband MLFMA
- Parallelization

- Examples from various domains

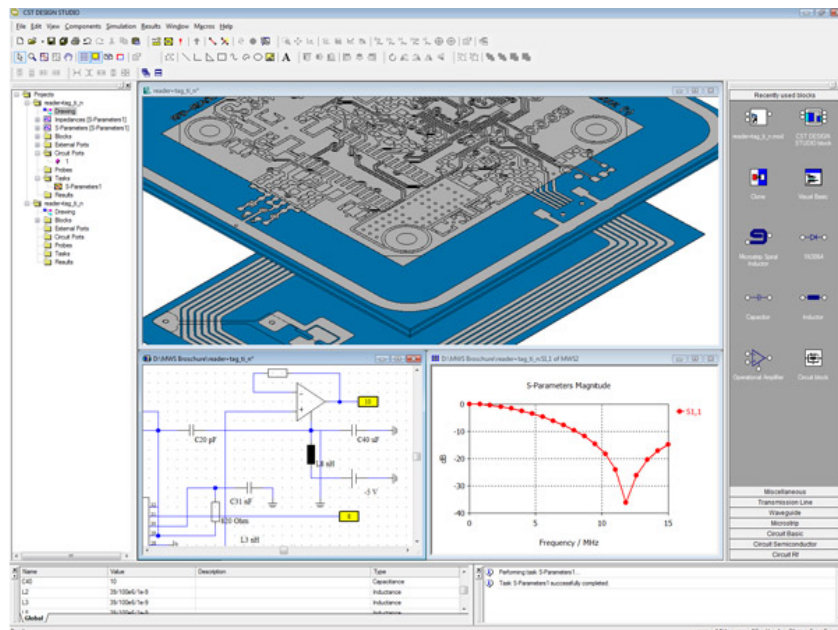
- Future challenges

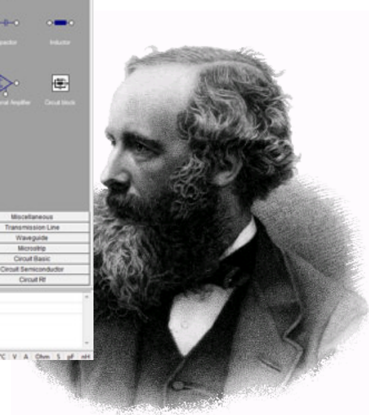
* in cooperation with the

+ Dept. of Electrical and Computer Engineering, University of Michigan, USA
+ Antennas and EMC Lab (LACE), Electronics Department, Politecnico di Torino, Torino, Italy



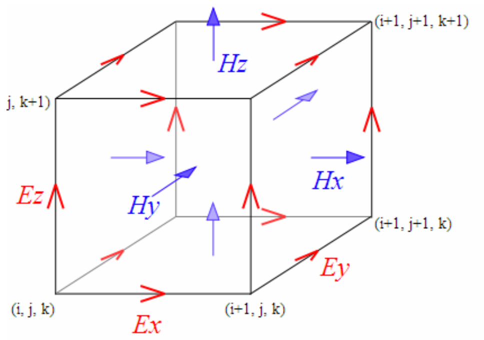

POWER
WLAN
WWAN
LAN


S-Parameters Magnitude
Frequency / MHz



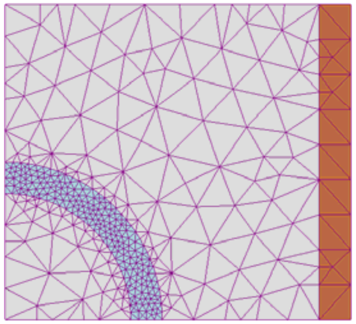
FDTD

(Finite Difference Time Domain)



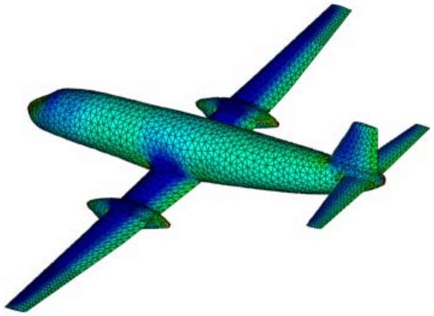
FEM

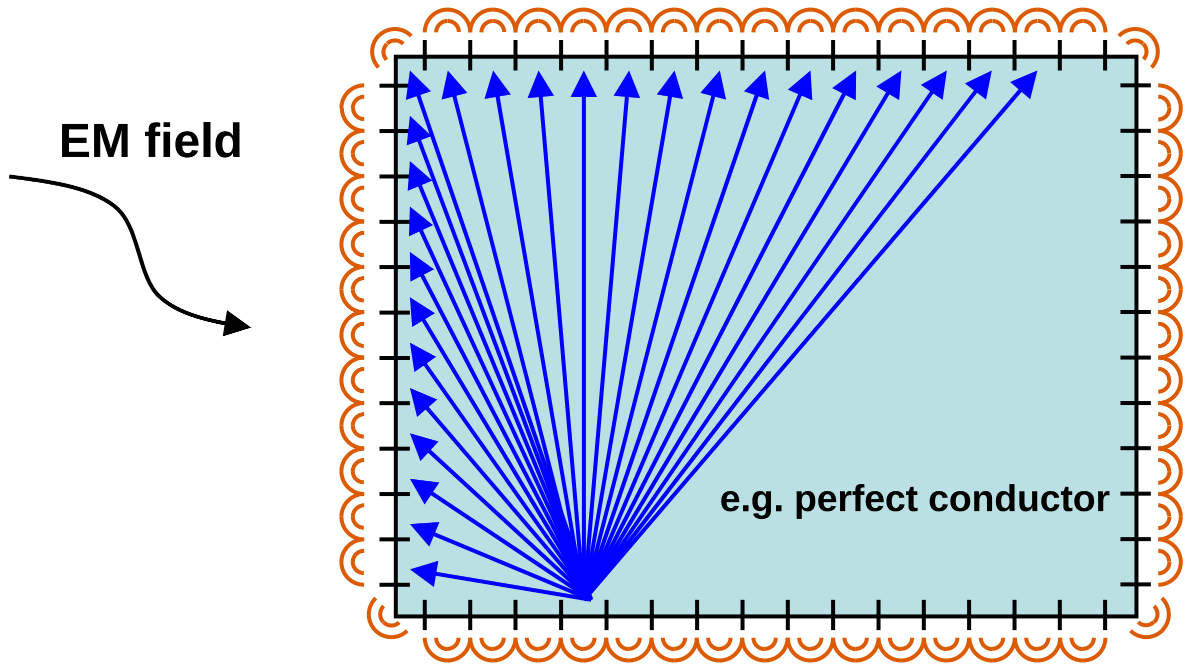
(Finite Elements)

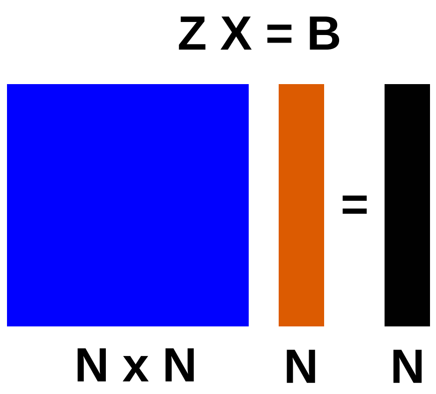


BIE

(Boundary Integral Equation)



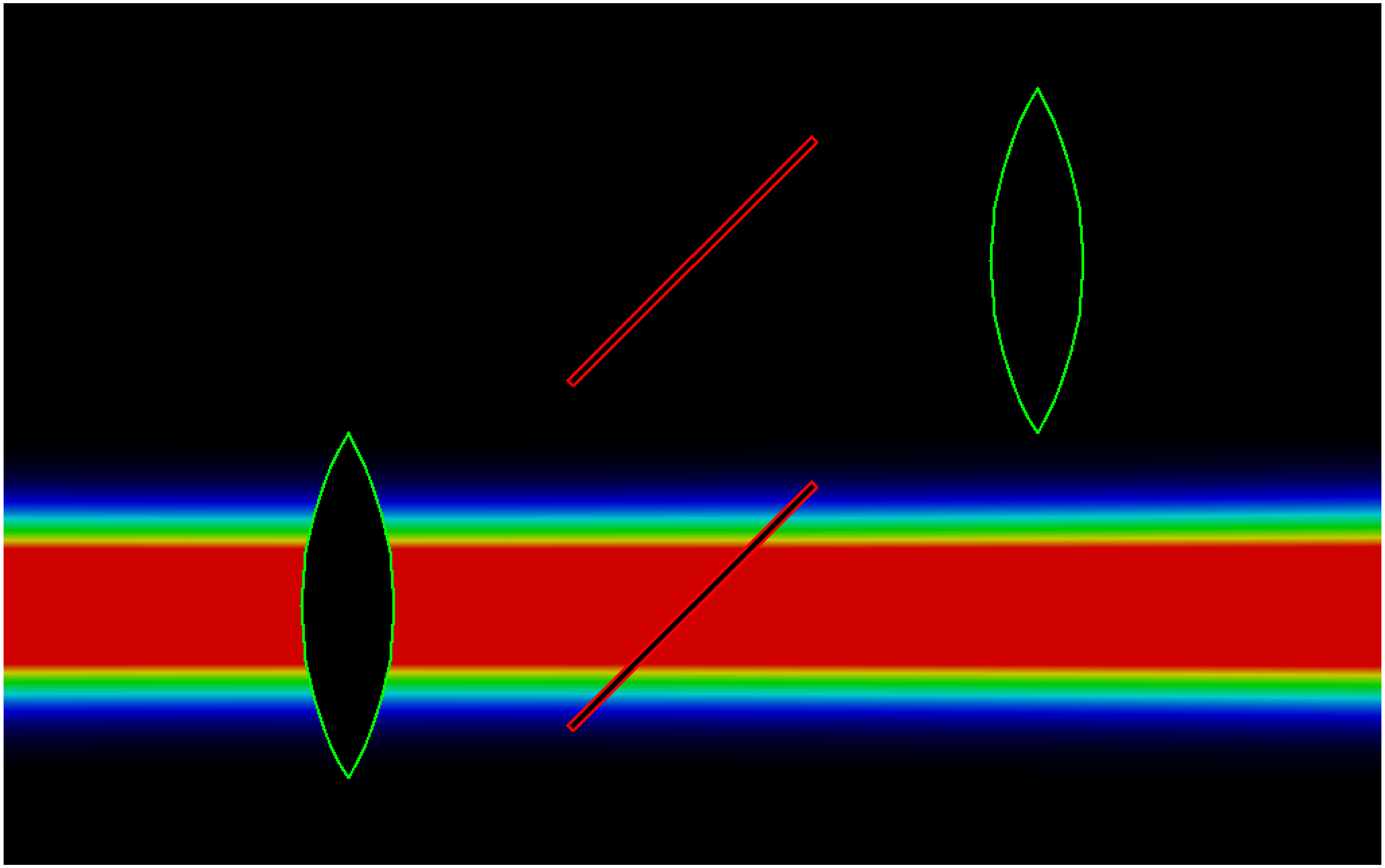




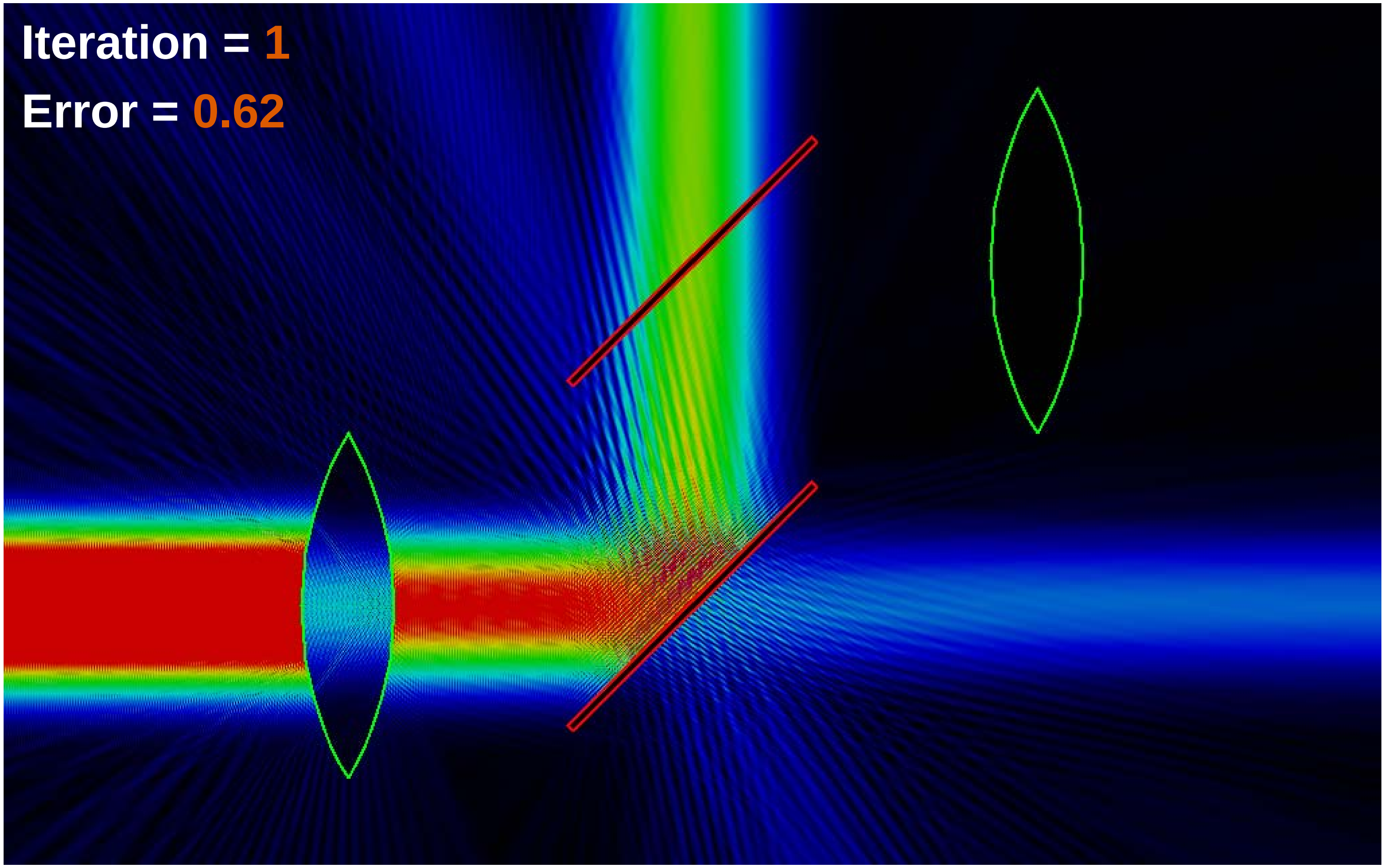
Linear set of equations with system matrix Z

- **Direct methods = O(N³) CPU time and O(N²) memory**
- **Iterative solution reduces CPU time to $O(N_{it}N^2)$ with $N_{it} << N$**

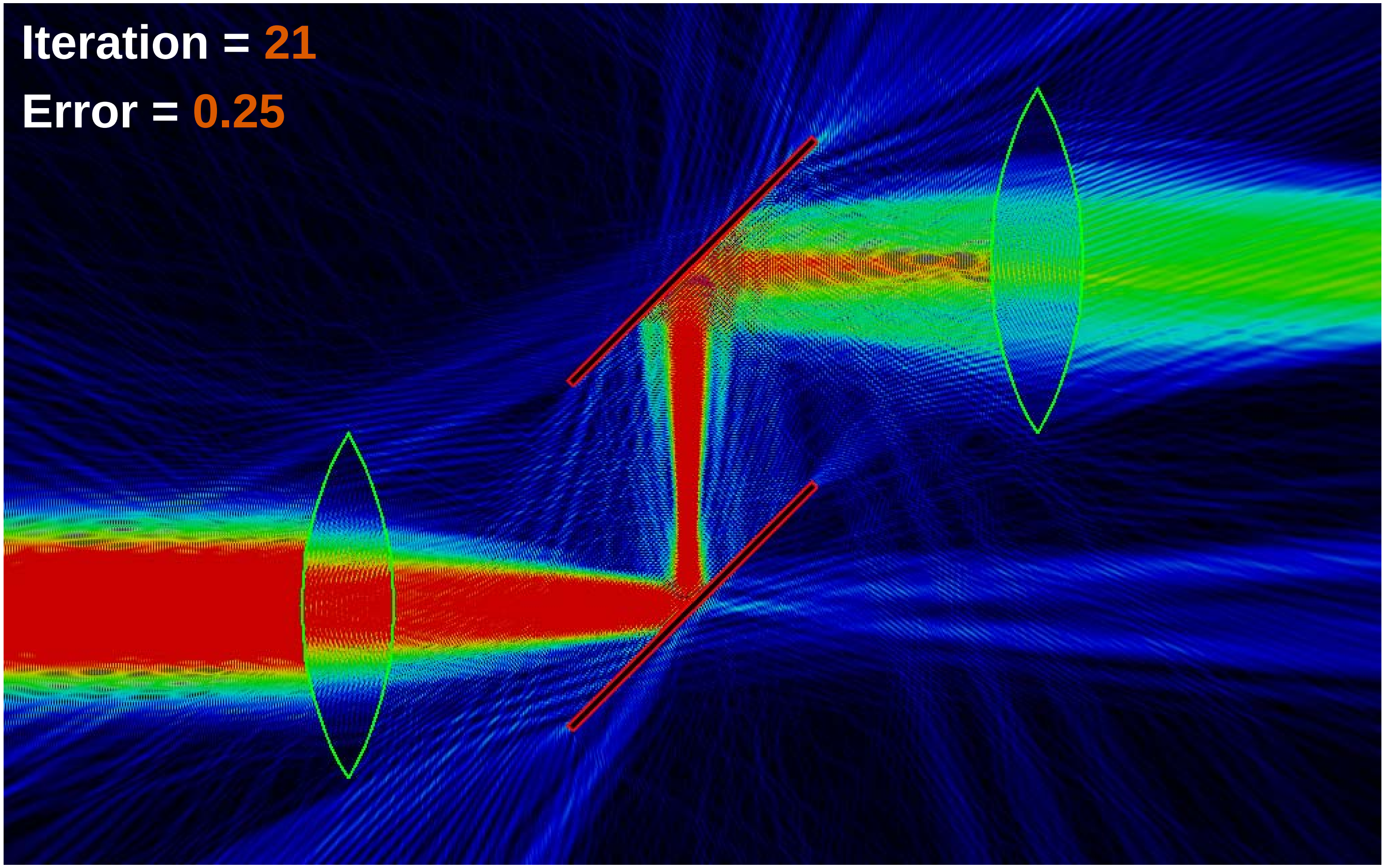
Iterative solution: example



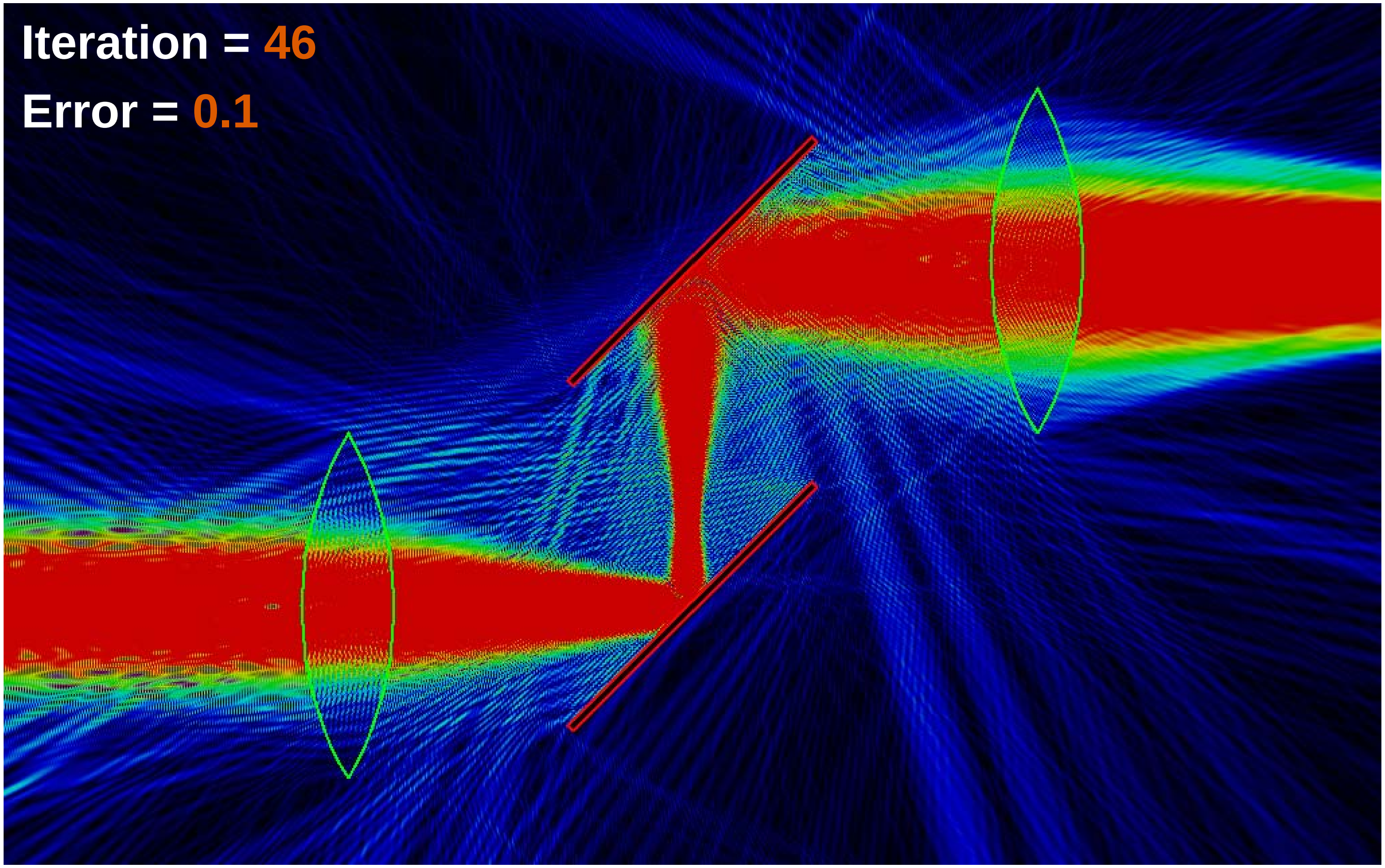
Iterative solution: example



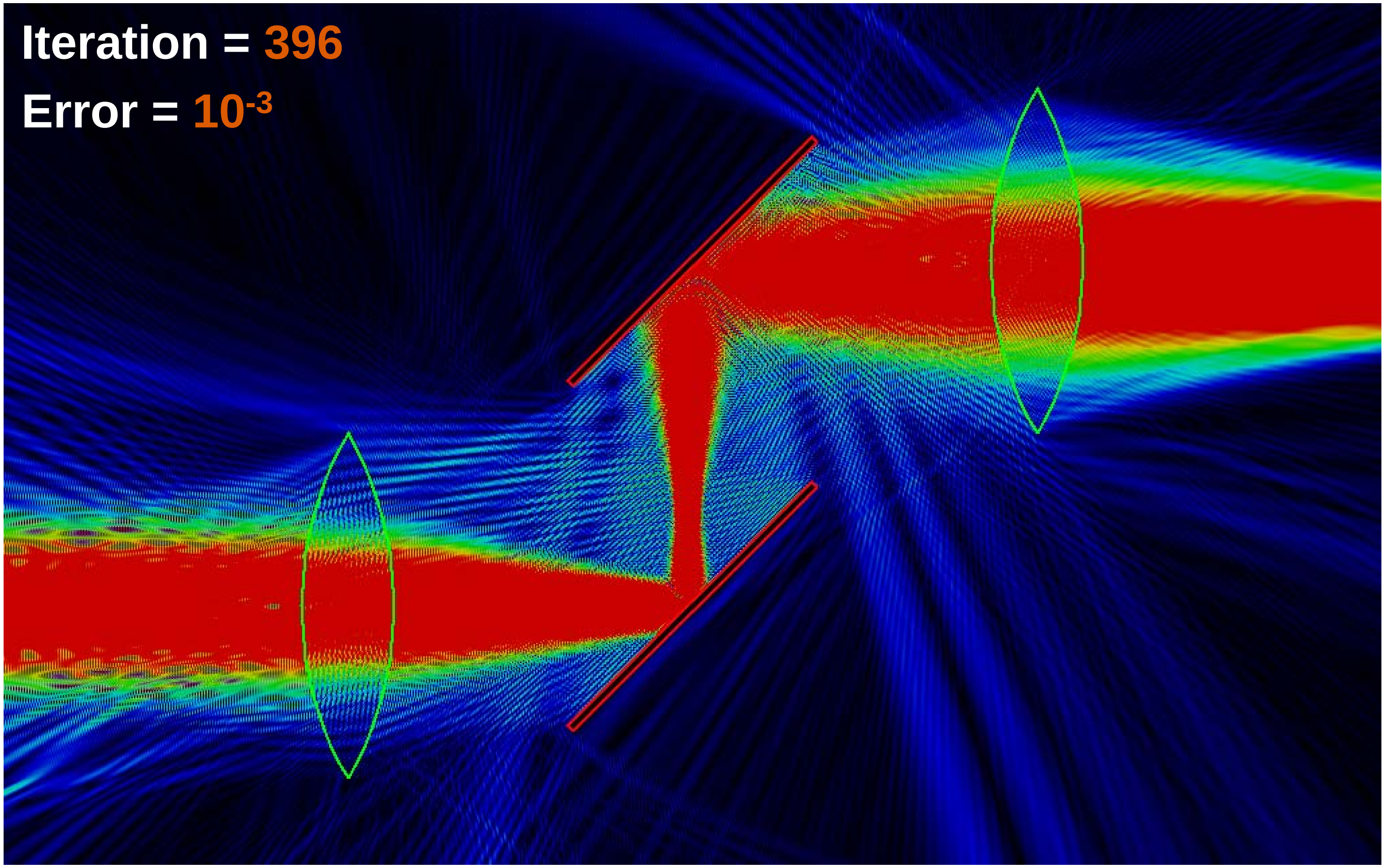
Iterative solution: example



Iterative solution: example



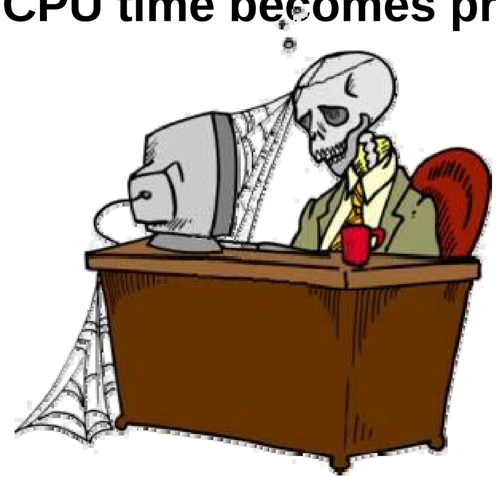
Iterative solution: example



Fast techniques

However, for large problems

- **very large amounts of memory are needed**
- **CPU time becomes prohibitive**

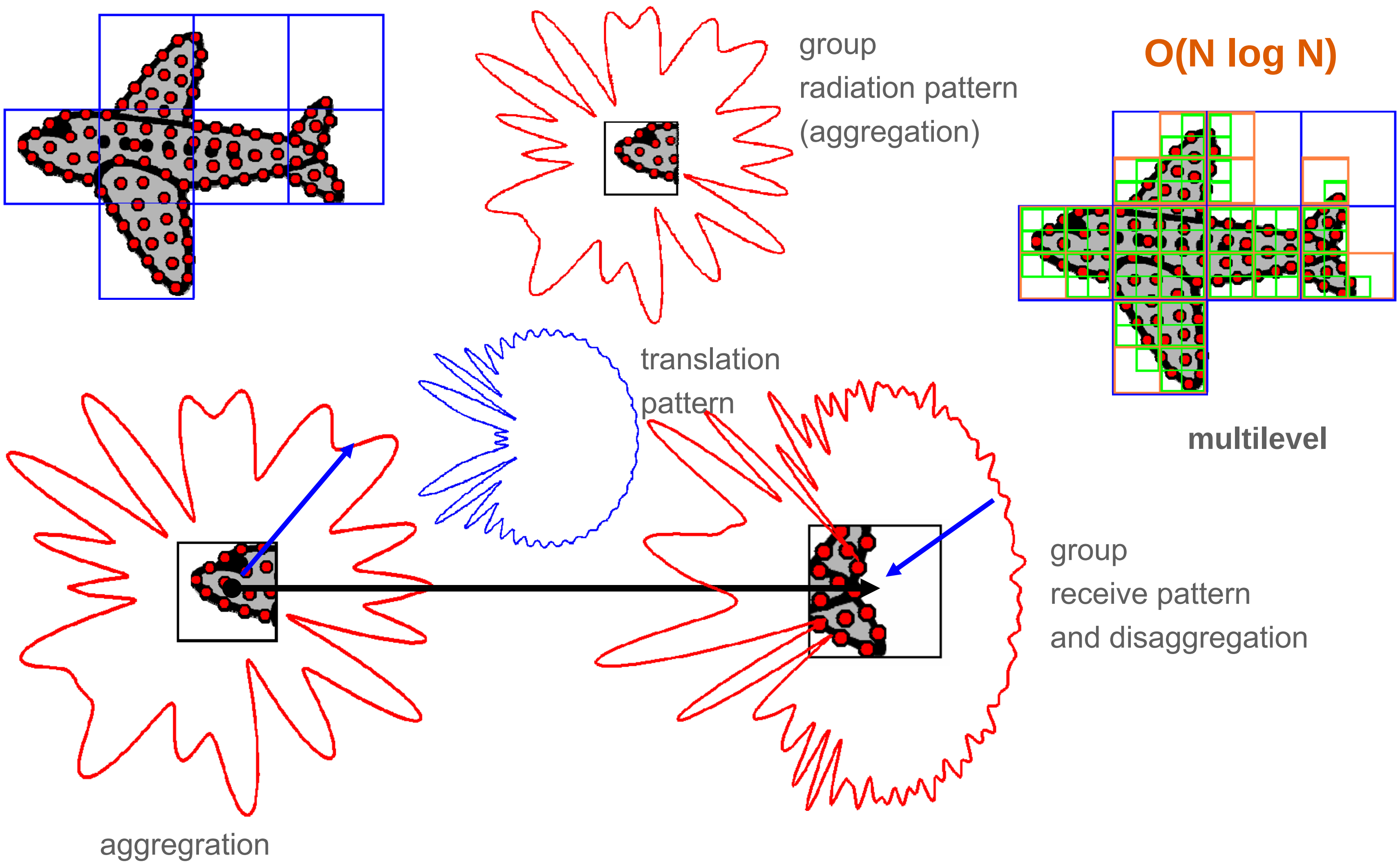


Solution: much improved iterative technique

- each iteration = a matrix-vector products **$Z*X_{guess}$**
- classical matrix-vector product is **$O(N^2)$**
- much faster: Multilevel Fast Multipole Algorithm (**MLFMA**)

⇨ **$O(N \log N)$!!** ☺

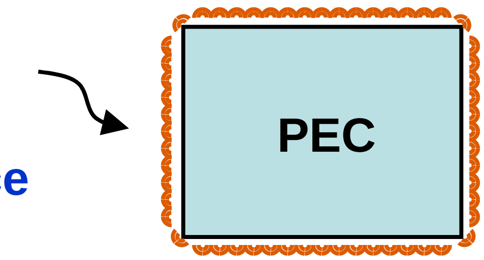
The physics of MLFMA



Problems and challenges

- keep the number of iterations small i.e. the iterative updates must converge as fast as possible to the actual solution → preconditioning P Z X = P B
- make it work over a large frequency range i.e. make it broadband: DC to mm-wave → non-directive stable plane wave MLFMA
- *note: make it work for structures with small details!*
- solve problems that are very large with respect to the wavelength → parallelize the MLFMA

Integral equations for the surface current on a PEC



$n \times e_i = - n \times e_{sc}$ or $n \times e_{tot} = 0$ n: normal to the surface

$n \times e_{sc} = T[\, j_{surf} \, , \rho_{surf}]$ ➡ **EFIE**

$= - n \times j\omega A\,(j_{surf}) - n \times \nabla\phi\,(\rho_{surf})$ with $\rho_{surf} = -\nabla . j_{surf} / j\omega$

$n \times h_i + n \times h_{sc} = j_{surf}$

$n \times h_{sc} = K[\, j_{surf} \,]$ ➡ **MFIE**

$= n \times \nabla \times A\,(j_{surf})$

- operator $T(\omega)$ becomes unbounded at low frequencies (or fine mesh)
- operator K remains bounded at low frequencies
- Calderón identity: $T^2 + K^2 = 1/4$
- operator T^2 remains bounded at low frequencies

Relevant integral equations for the surface current on a perfect conductor

$$\hat{n} \times e^{i} = - T[j](r)$$
$$= -\frac{1}{j\omega\epsilon}\hat{n} \times \int_{\Gamma} \nabla \frac{e^{-jkR}}{4\pi R} \nabla' \cdot j(r')\, dS'$$
$$+ 1/2\; j\omega\mu \hat{n} \times \int_{\Gamma} \frac{e^{-jkR}}{4\pi R} j(r')\, dS',$$

tangential comp. of incident field

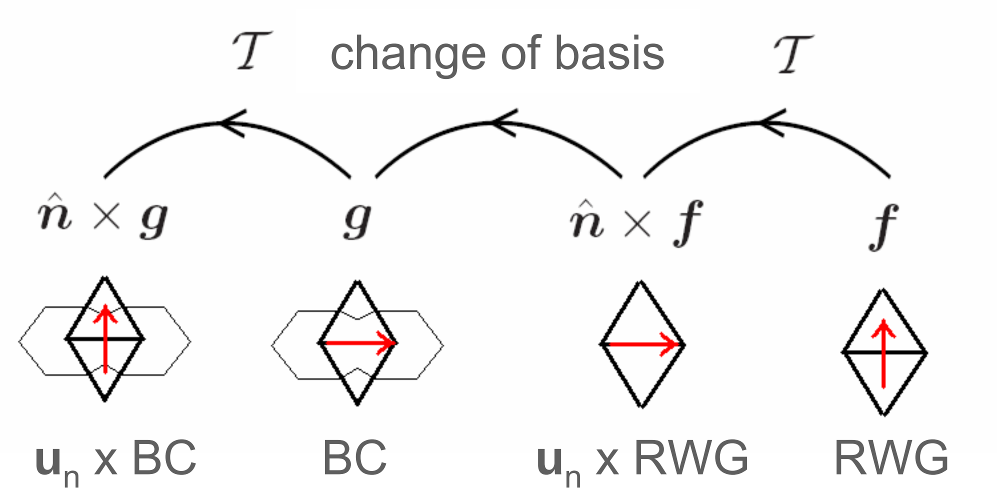
$$\hat{n} \times h^{i}(r) = \left\{\frac{1}{2} + K\right\}[j](r)$$
$$= \frac{1}{2} j(r) - \hat{n} \times \frac{1}{4\pi} \int_{\Gamma} \nabla \frac{e^{-jkR}}{R} j(r')\, dS'$$

tangential comp. of incident field

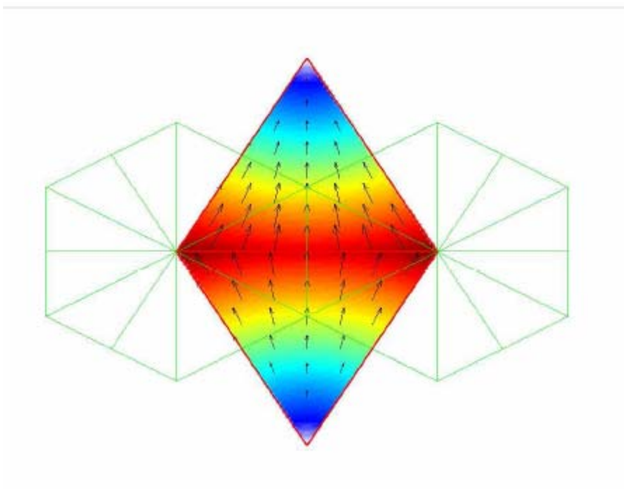
- **operator T(ω) becomes unbounded at low frequencies (or fine mesh)**
- **operator K remains bounded at low frequencies**
- **Calderón identity: $T^2 + K^2 = 1/4$**
- **operator T^2 remains bounded at low frequencies**

- **Preconditioning of EFIE: $T[n \times e_i] = -T[n \times e_{sc}] = T^2[j_{surf}, \rho_{surf}]$**

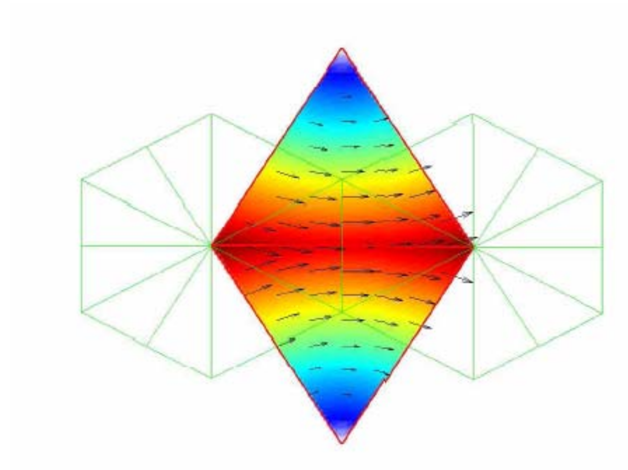
How to discretize T^2 such that spectral properties remain?



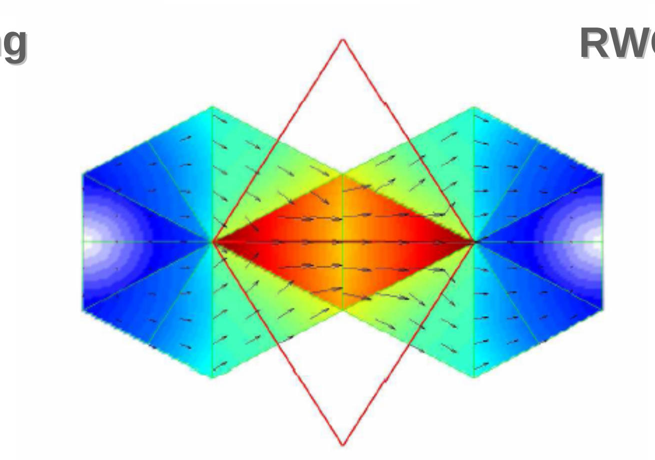
Relevant basis functions (1)



RWG div-conforming

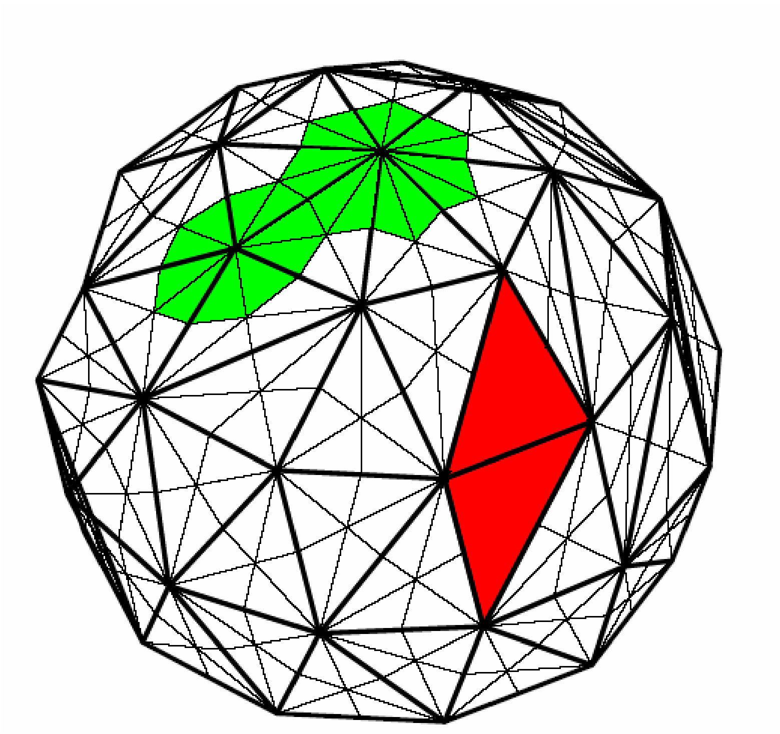


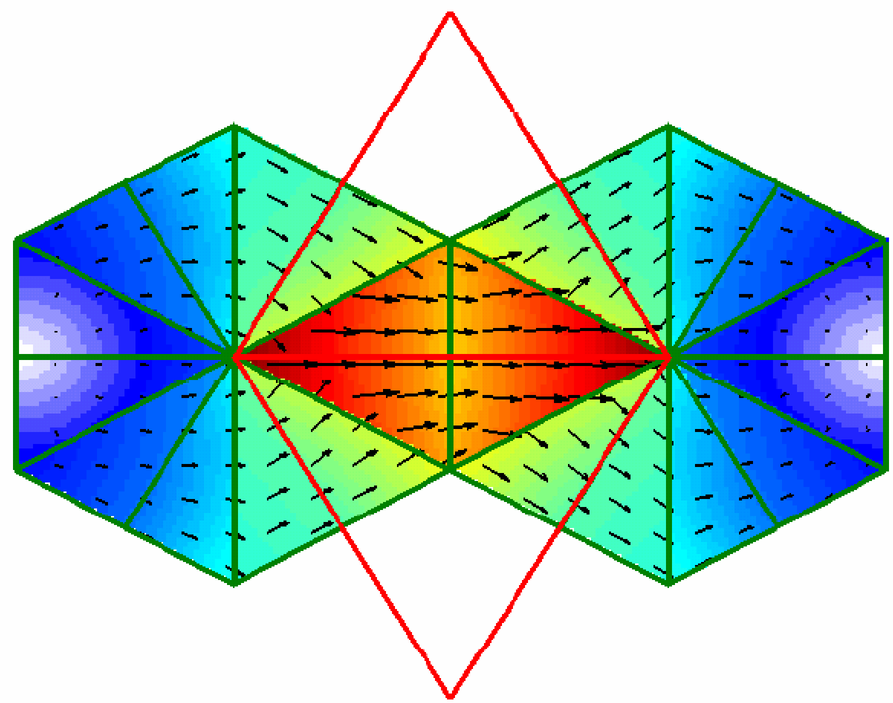
RWG curl-conforming



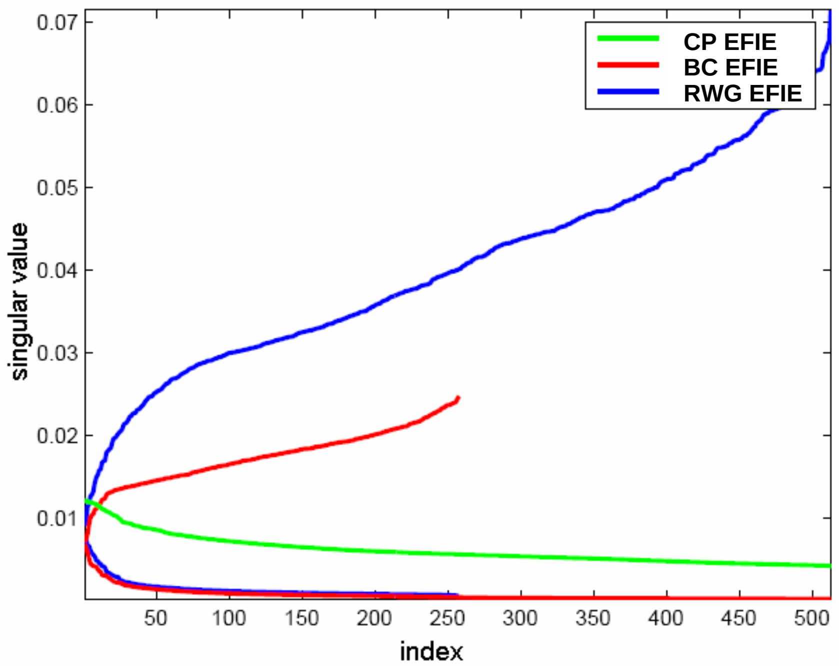
Buffa-Christiansen quasi curl-conforming

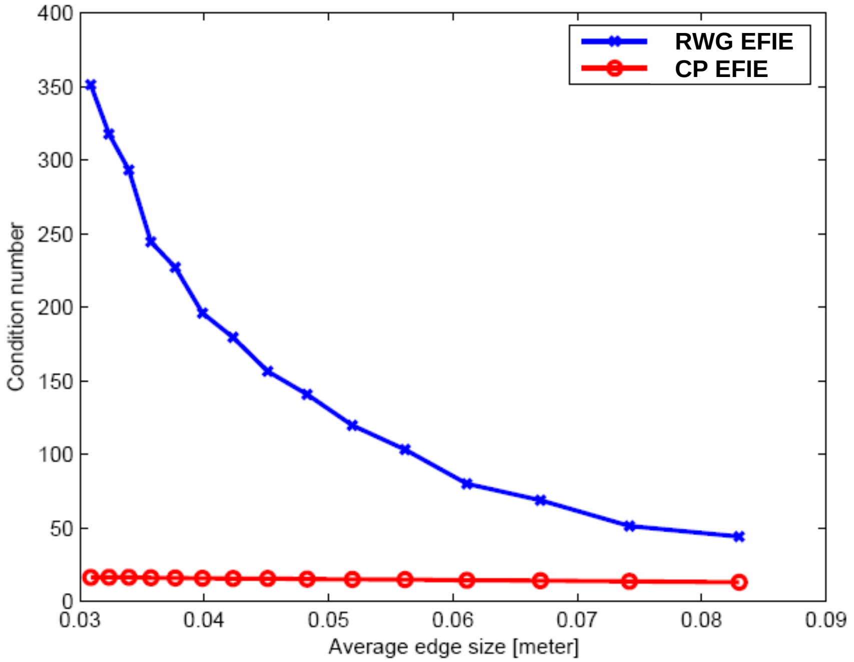
- **Buffa-Christiaensen (BC) basis functions**





- **Effect of Calderón preconditioning (CP)**

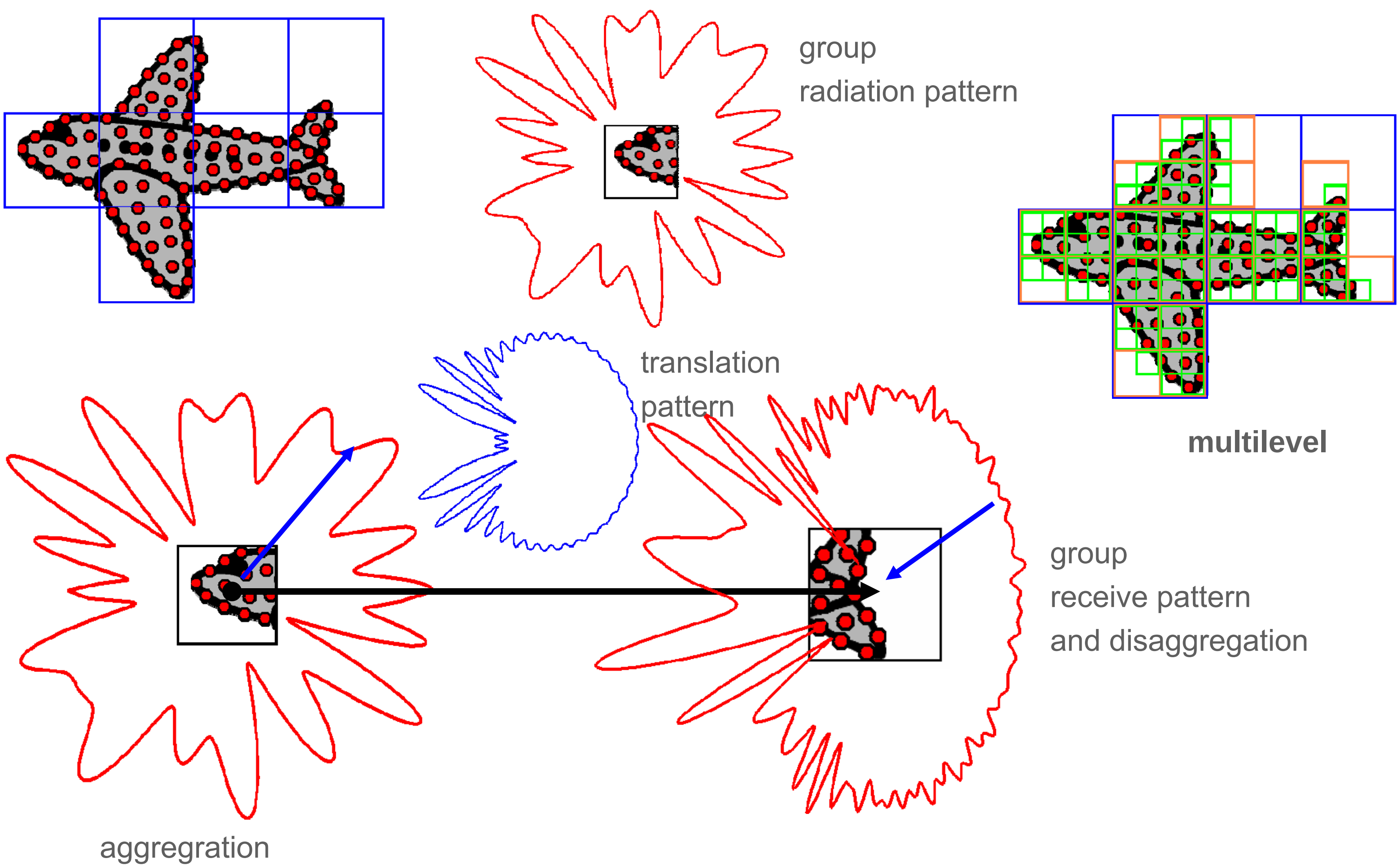




fixed number of mesh cells

increasing number of smaller mesh cells

The physics of MLFMA



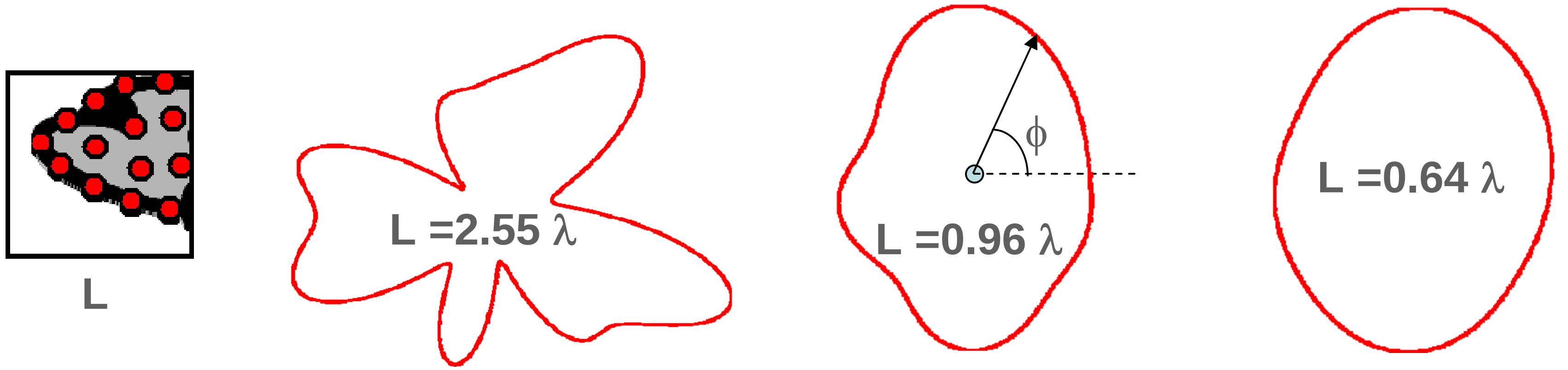
- **plane-wave based MLFMA breaks down at low frequencies**
  - Does not incorporate evanescent field information only propagating plane waves
- **existing solution**
  - Use multipole expansion at low frequencies (dipole, quadrupole, ...)
    - Non-diagonal translation matrices
    - Difficult to combine with MLFMA
- **new technique**

  Non-directive/analytical stable plane wave MLFMA (NSPWMLFMA)
    - Diagonal translation matrices
    - Easy to combine with MLFMA

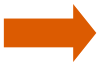
broadband MLFMA

- **plane-wave based MLFMA breaks down at low frequencies**
  - Does not incorporate evanescent field information only propagating plane waves
- **new technique**

  Non-directive/analytical stable plane wave MLFMA (NSPWMLFMA)



Plane-wave based MLFMA breaks down at low frequencies



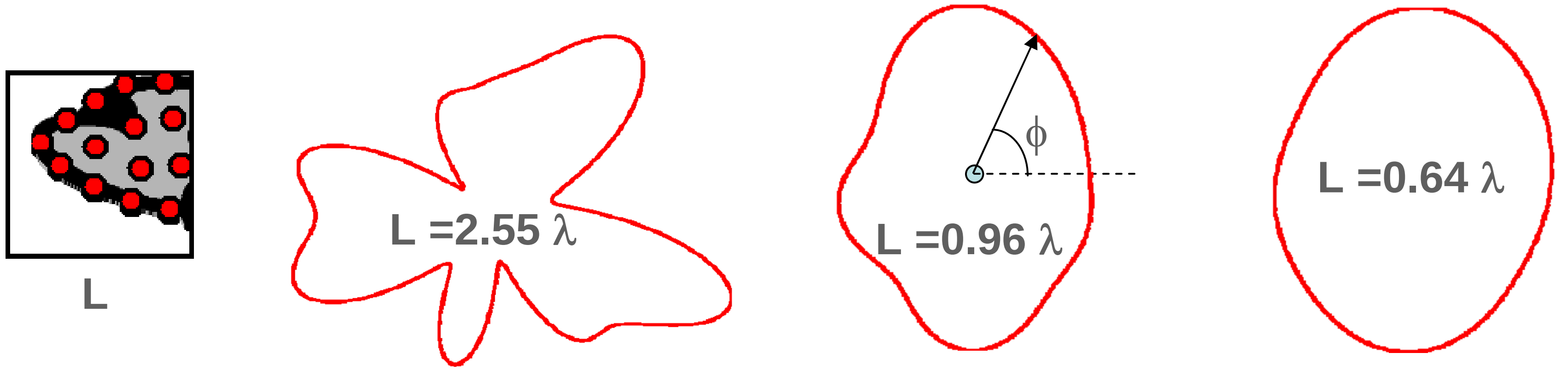
Does not incorporate evanescent field information

only propagating plane waves

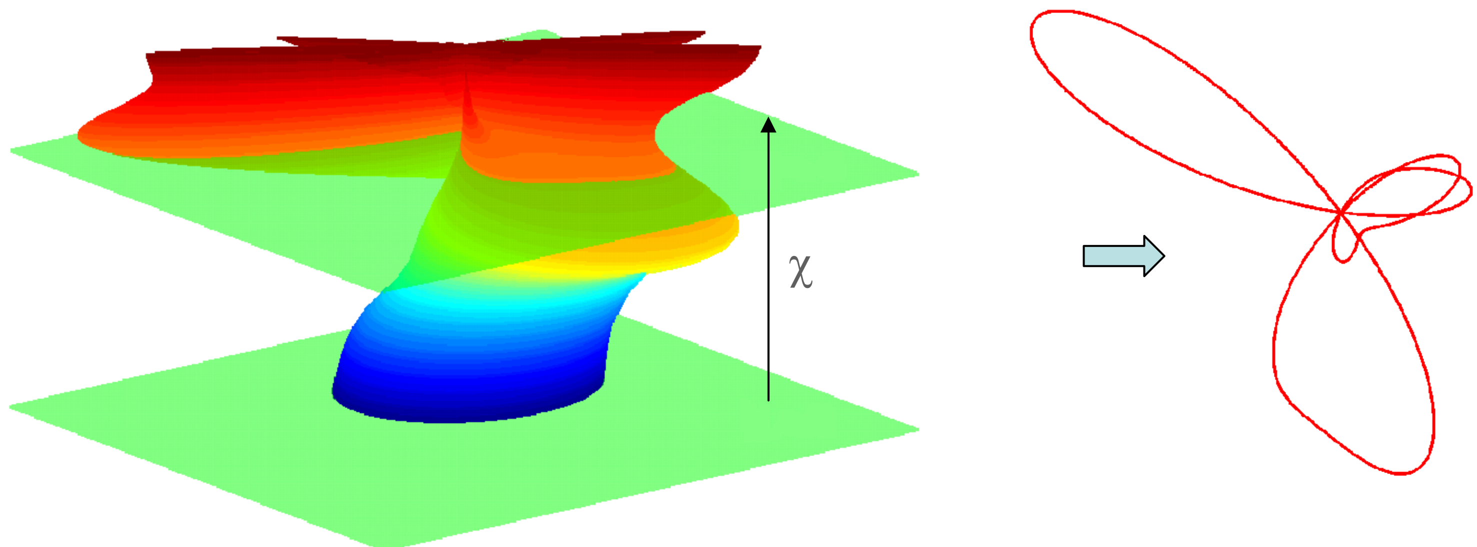
New technique

Non-directive/analytical stable plane wave MLFMA (NSPWMLFMA)

Complex plane radiation pattern



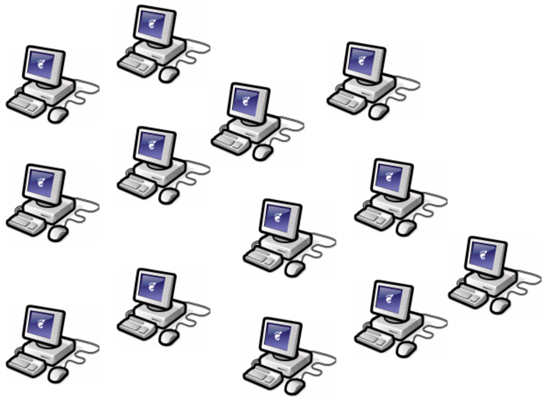
make ϕ complex: $\phi + j\chi$



Parallel MLFMA

classical O(N²)	MLFMA O(N log N)	Parallel MLFMA O(N log N)
N < 10 000 unknowns	N < 1 000 000	N > 100 000 000







1 λ

10 λ

100 λ

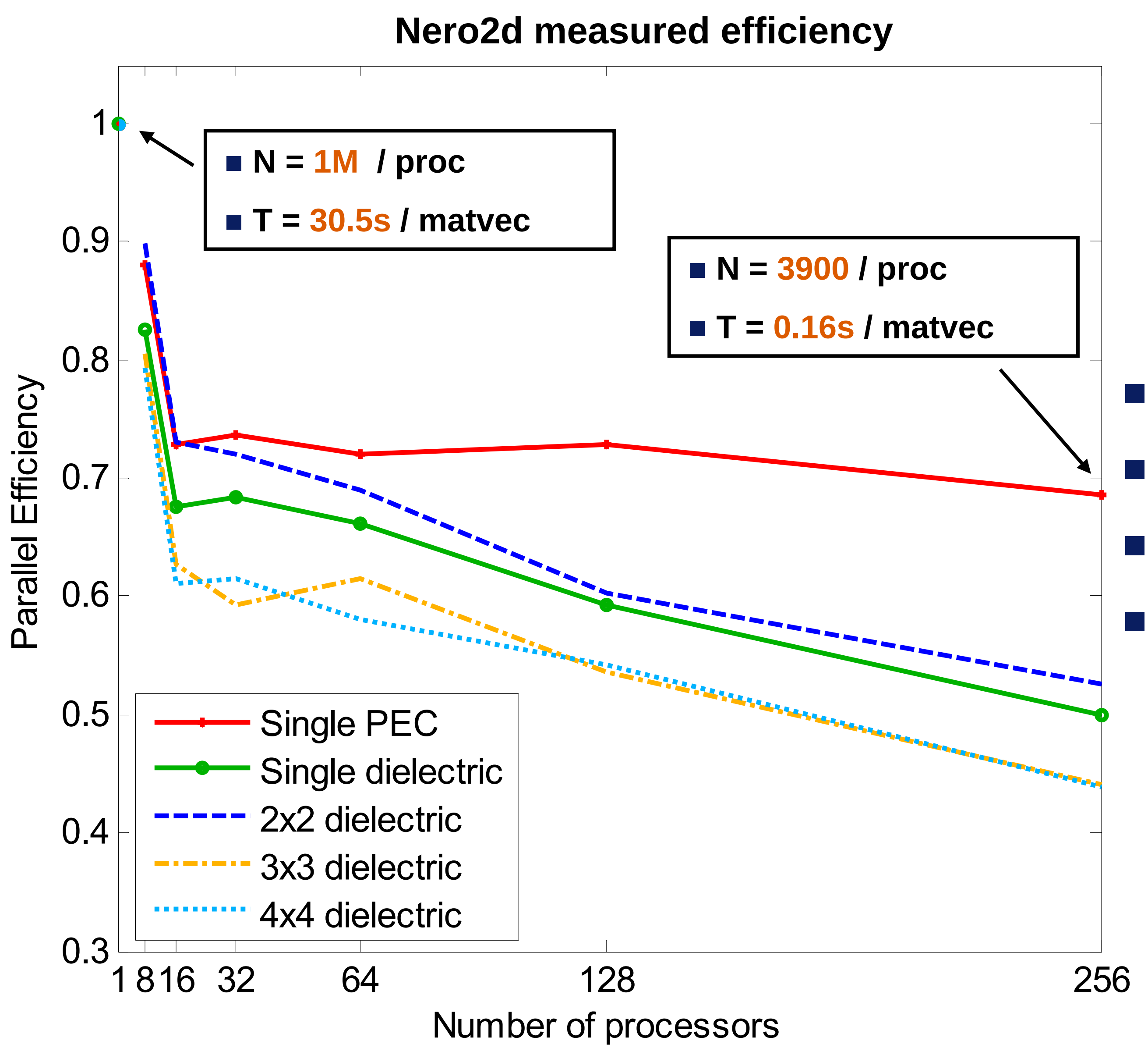
Parallelization of MLFMA

- **Previous efforts:**
  - **Simulation of large single 3D objects**
  - **Allows good load balancing**
  - **Synchronous algorithms (either communication or same type of calculation)**
- **Our efforts:**
  - **Simulation of complex geometries consisting of multiple objects**
  - **Difficult to obtain good load balancing**
  - **Asynchronous algorithm**

Parallelization of MLFMA

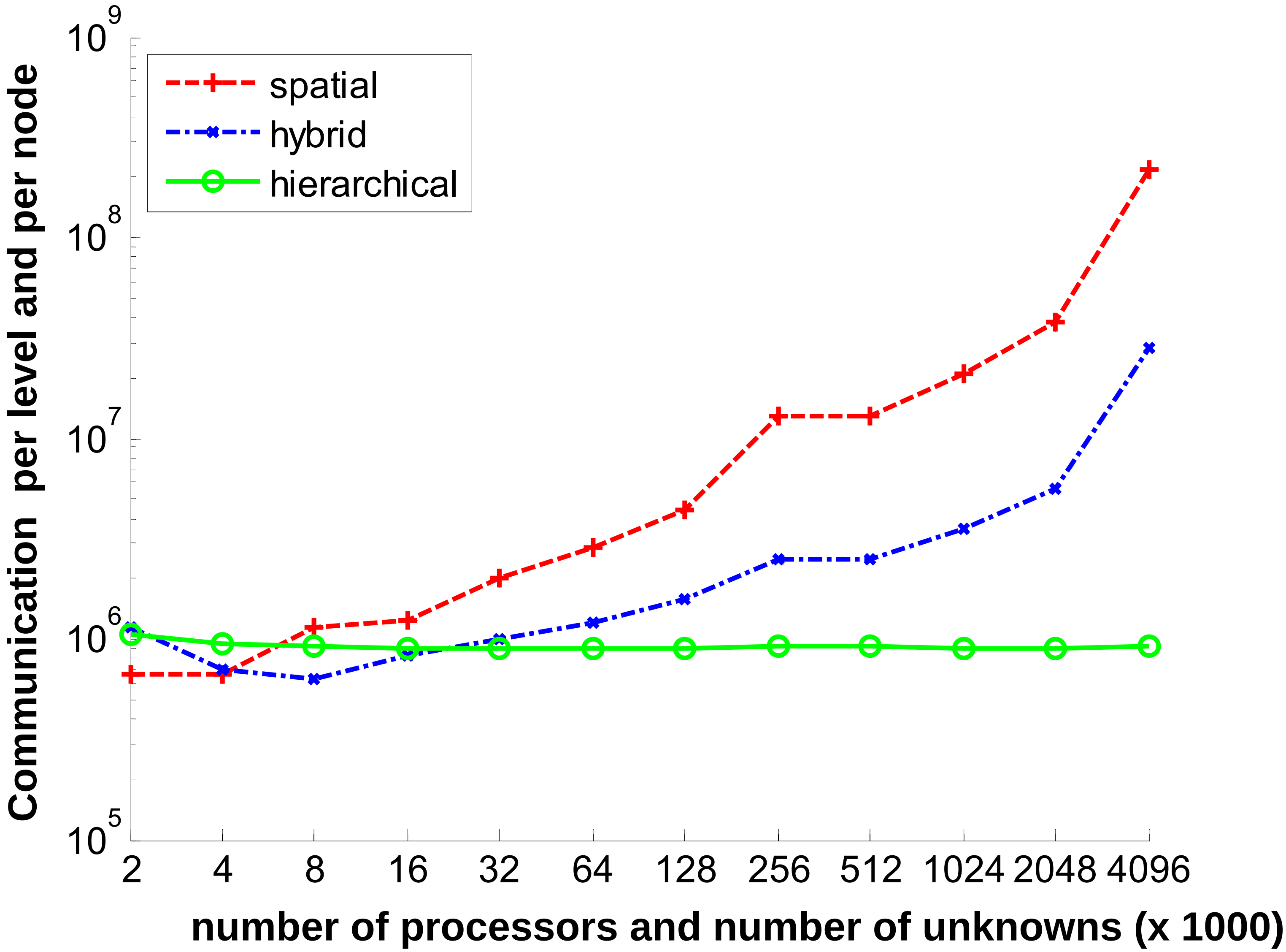
- **Previous efforts:**
  - **Simulation of large single 3D objects**
  - **Allows good load balancing**
  - **Synchronous algorithms (either communication or same type of calculation)**
  - **Fast interconnection environments (Infiniband)**
- **Our efforts:**
  - **Simulation of complex geometries consisting of multiple objects**
  - **Difficult to obtain good load balancing**
  - **Asynchronous algorithm**
  - **Focus both on parallel efficiency and parallel scalability using hierarchical partitioning**

Parallel Efficiency Infiniband



- Intel Xeon L5420
- 2 x 4 cores / machine
- 20 Gbit/s Infiniband
- Intel MPI

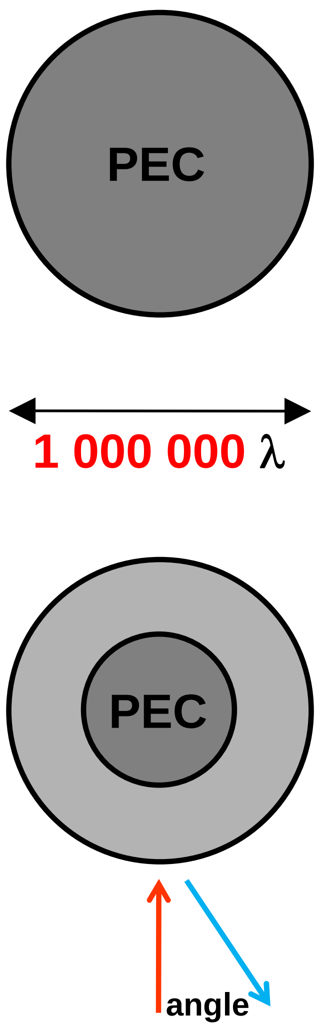
Parallel Scalability

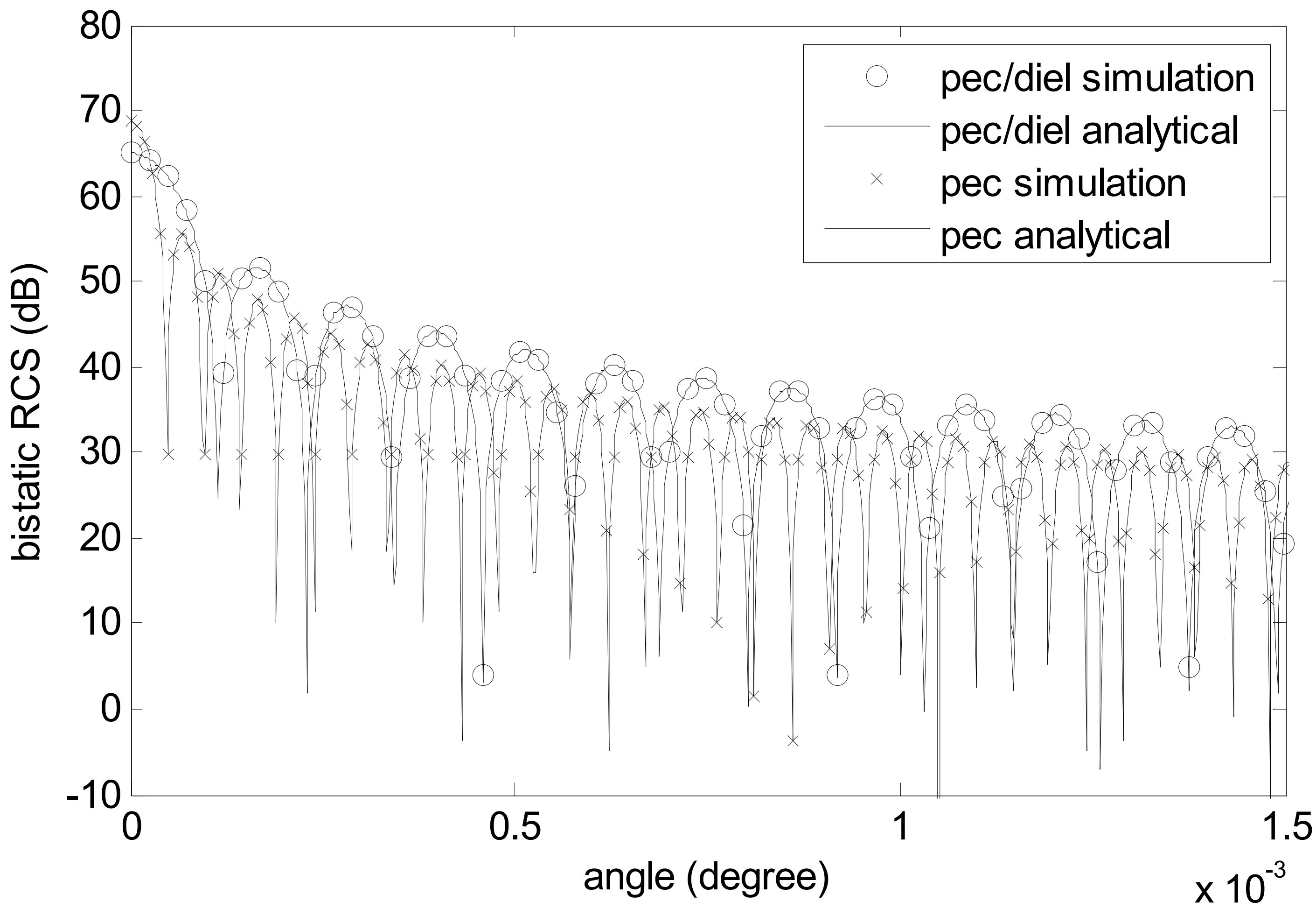


Some examples

- **proof of accuracy: analytical example – scattering by a cylinder**
- **scattering by an Airbus and by a “Thunderbird”**
- **indoor propagation**
- **shielding**
- **artificial media**
- **lens systems**
- **Cassegrain antenna**

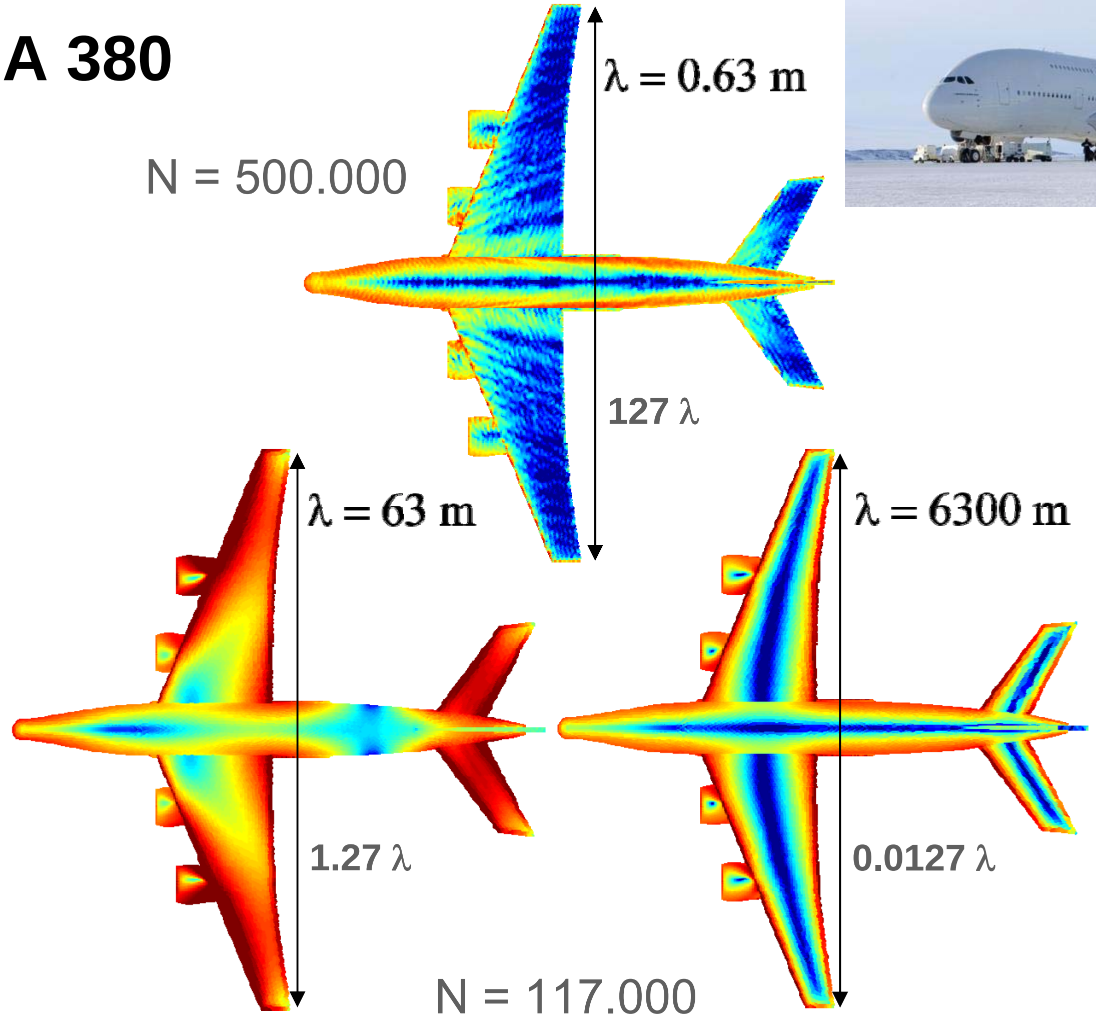
Is such a method “really” accurate

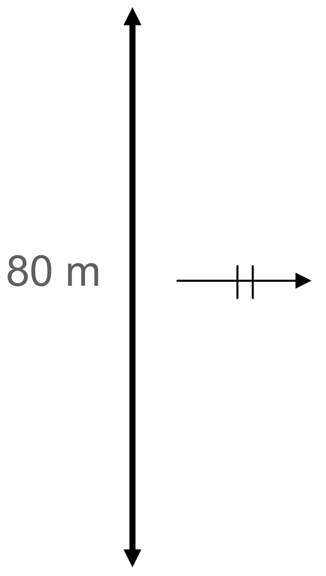




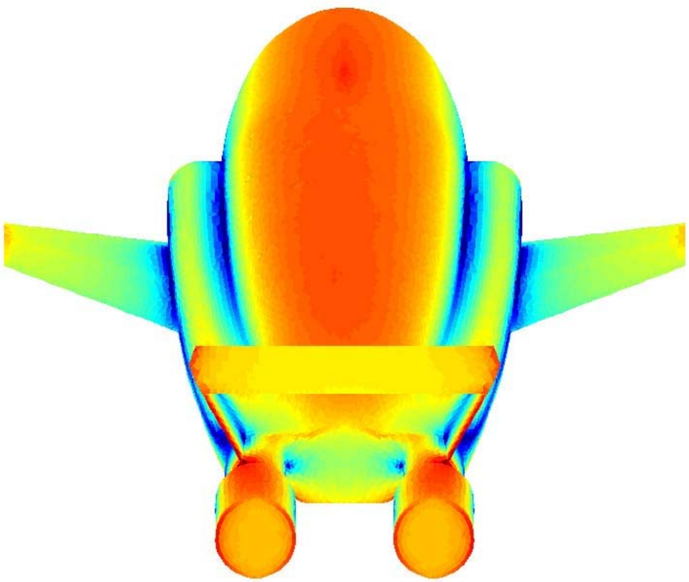
Broadband 3D scattering

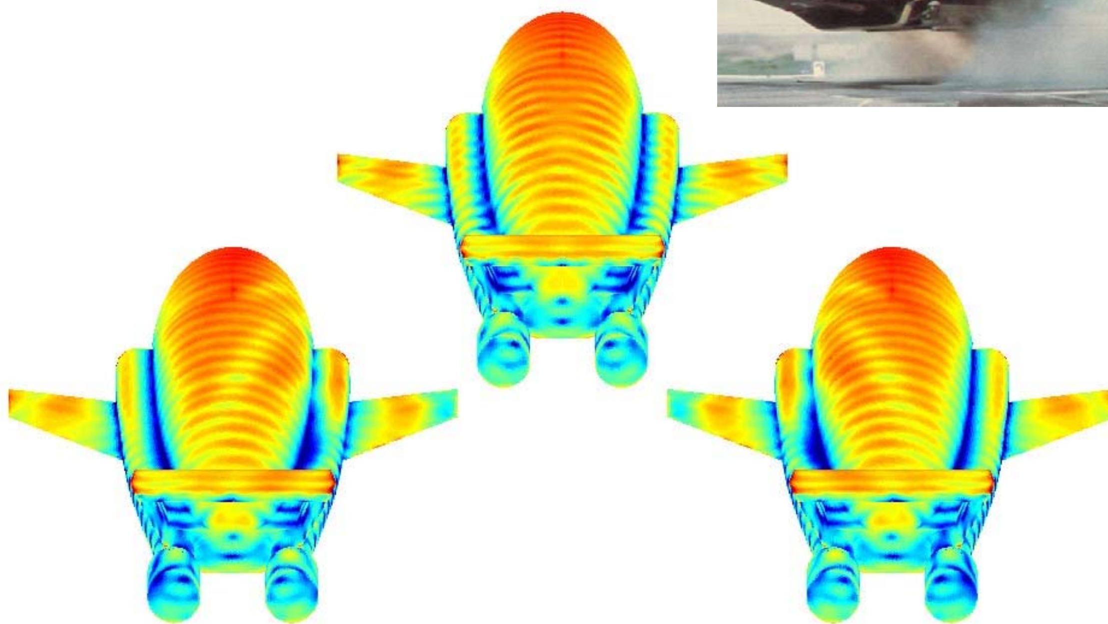
- **Airbus A 380**





- **LF and HF Thunderbird 2 (TB)**



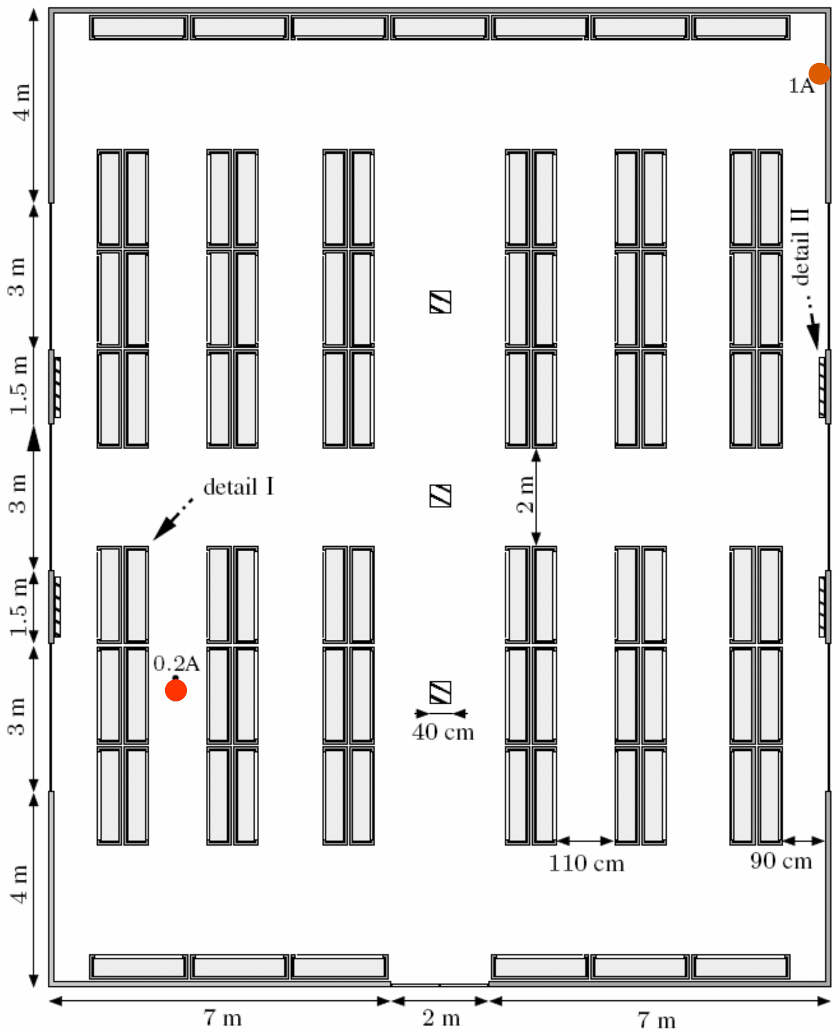


Length TB: 0.014 λ
N = 101.466
21 iterations
Accuracy: 10^{-3}
20s per iteration
12 AMD Opteron 270 processors

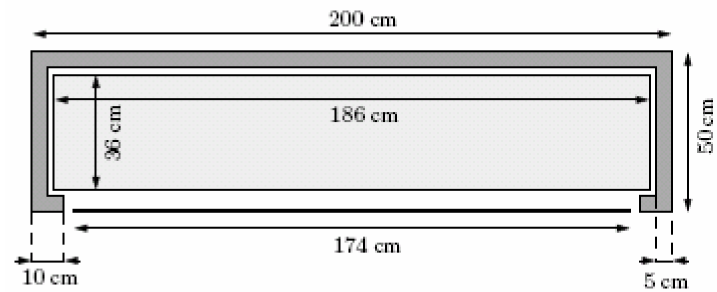
Length TB: 15 λ
N = 1.025.559, 1.2GByte
28 iterations
Accuracy: 10^{-3}
28s per iteration
20 AMD Opteron 270 processors

Library 20m x 16m

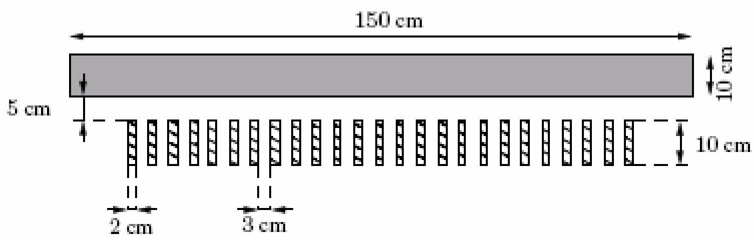
- **2 sources @ 60 GHz i.e. wavelength 0.5cm**



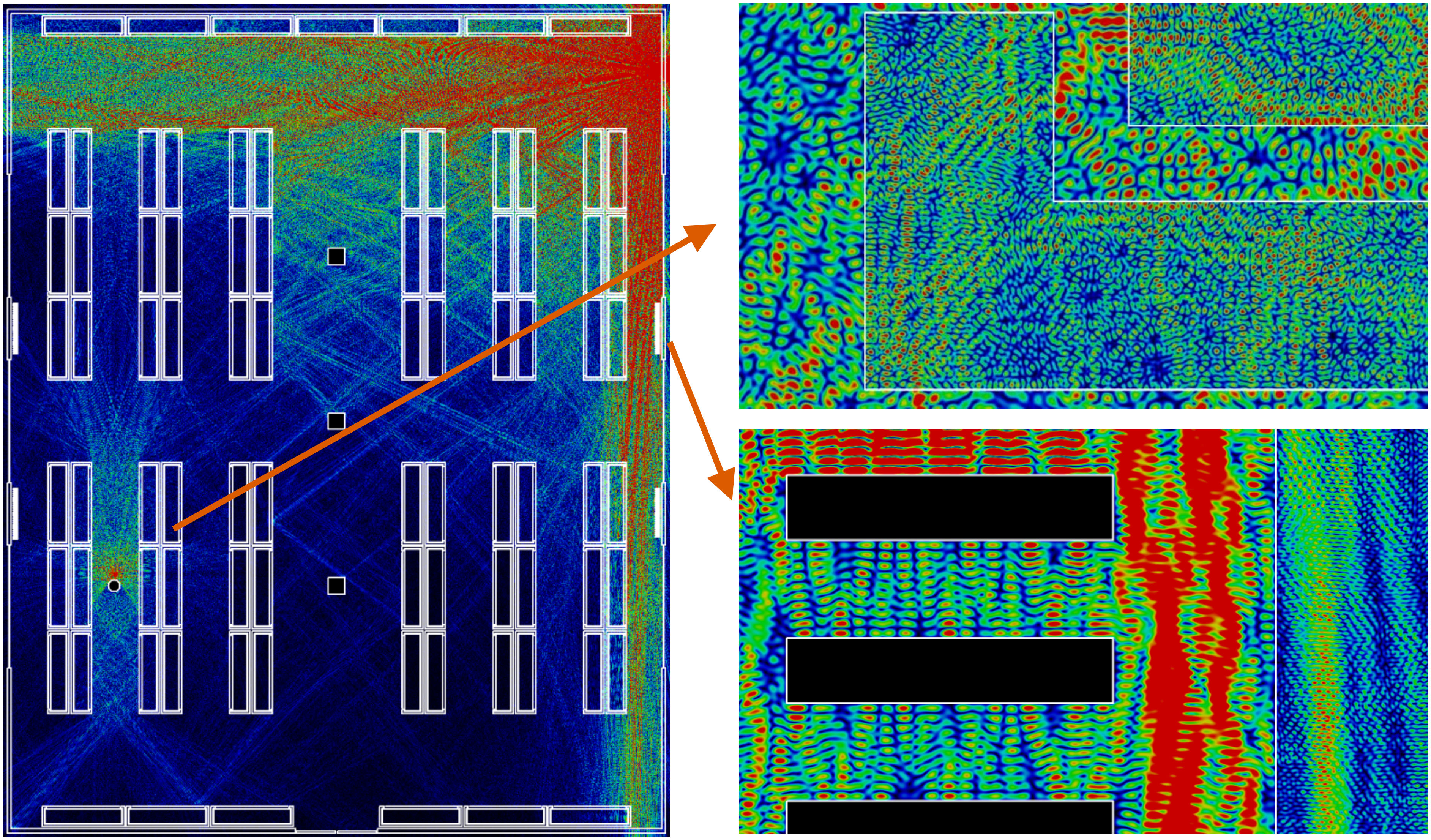
- **Detail 1**



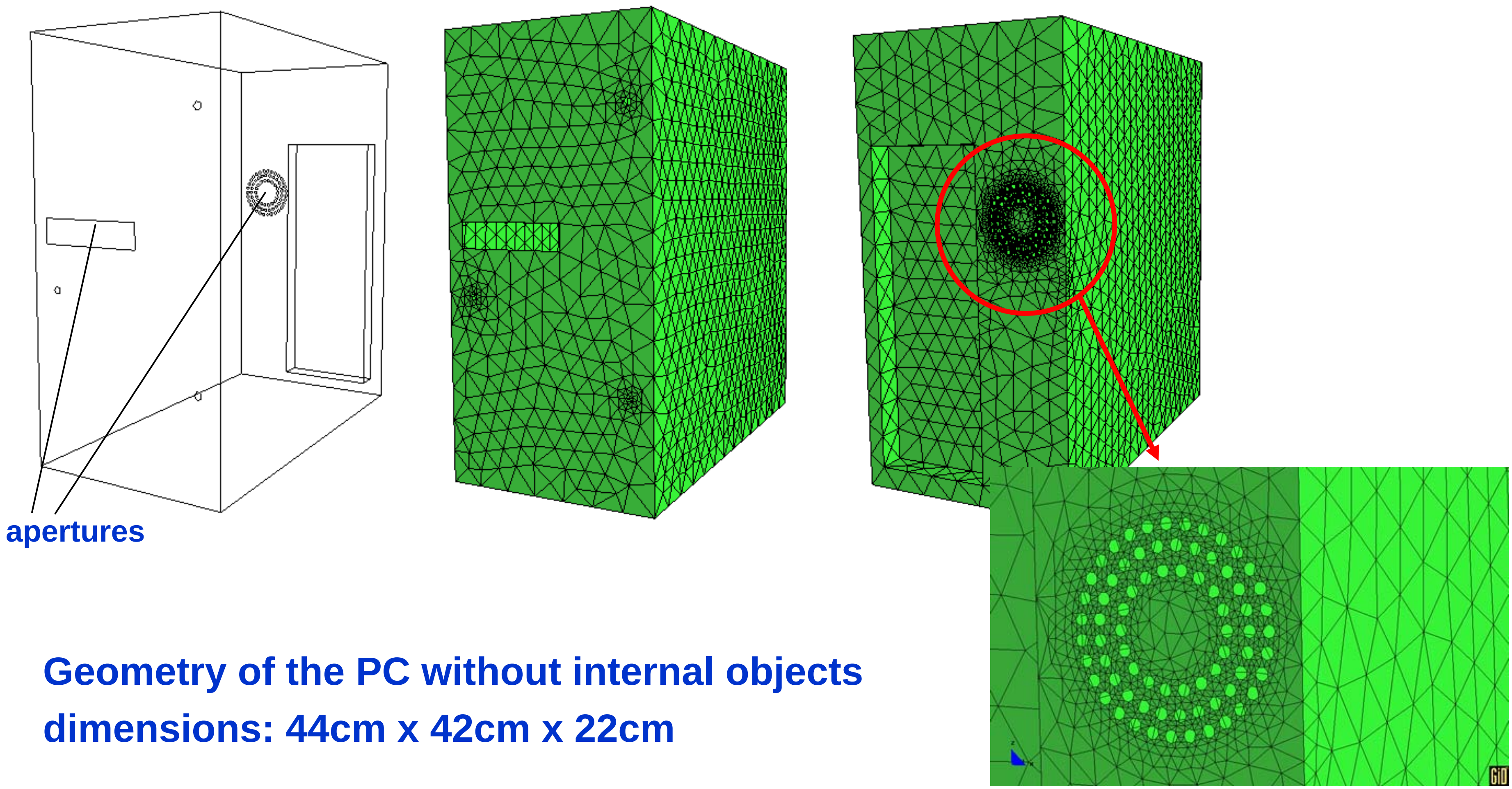
- **Detail 2**



Indoor propagation problem



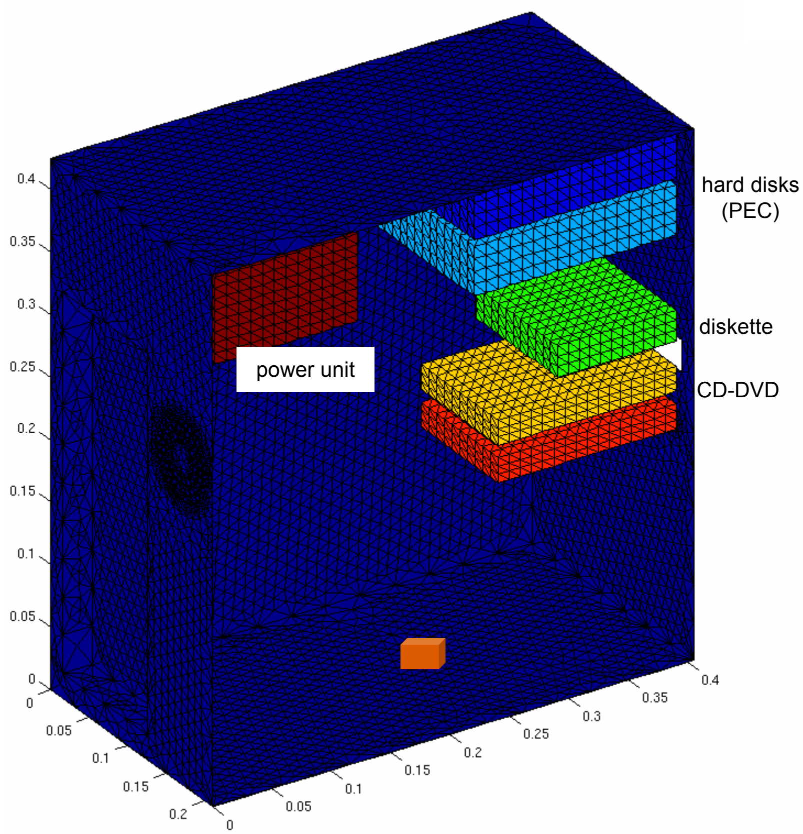
Personal Computer

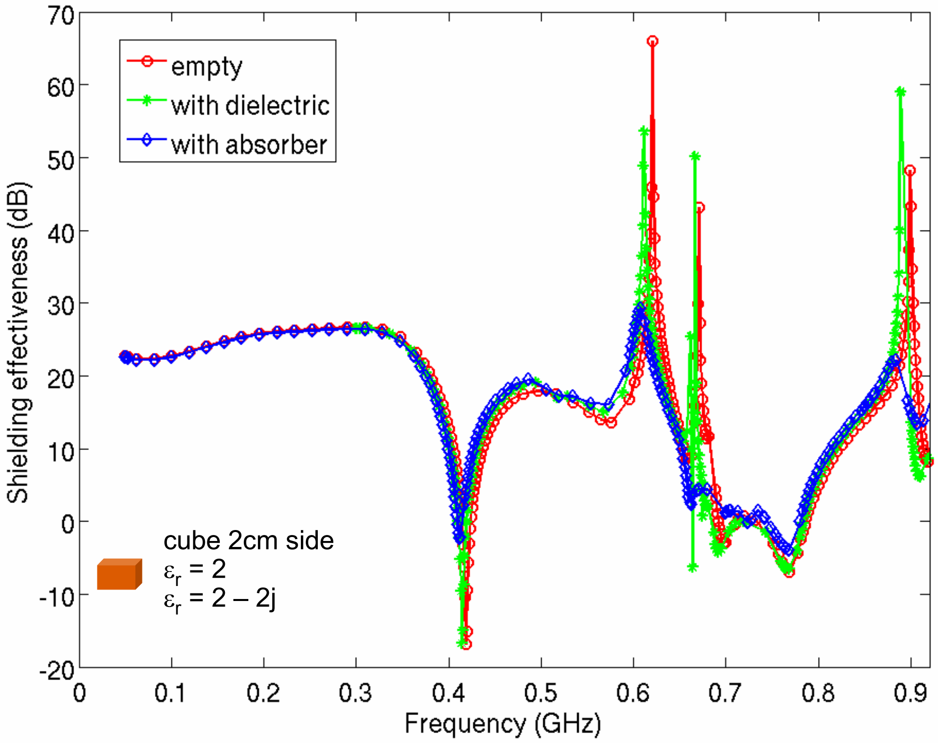


Geometry of the PC without internal objects

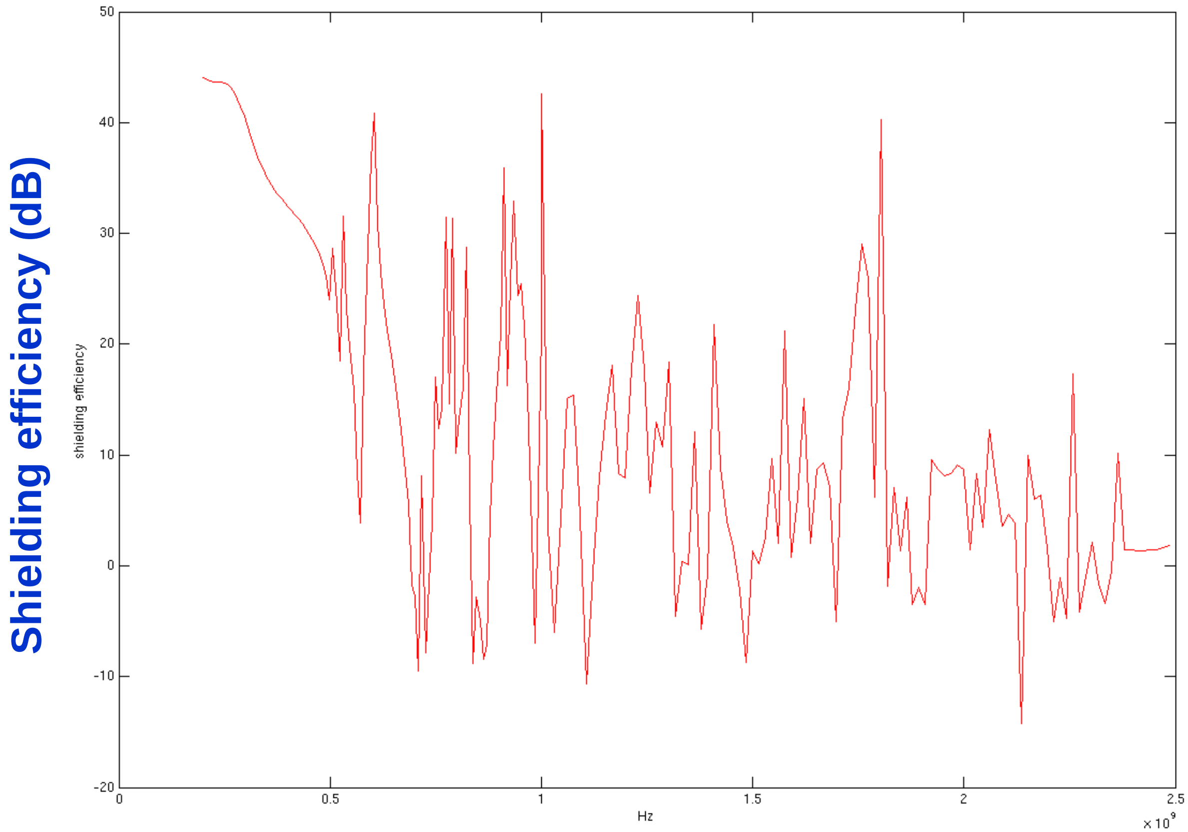
dimensions: 44cm x 42cm x 22cm

EMC Shielding

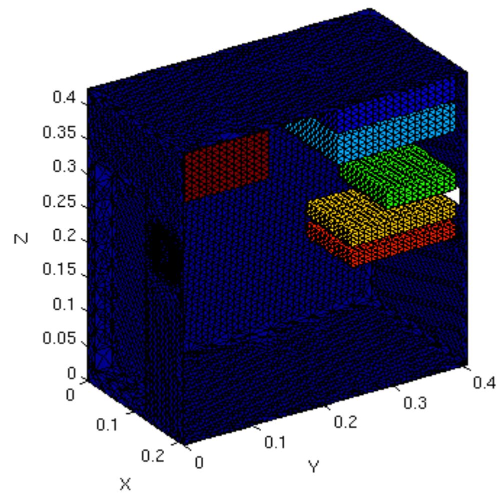




Shielding cont.



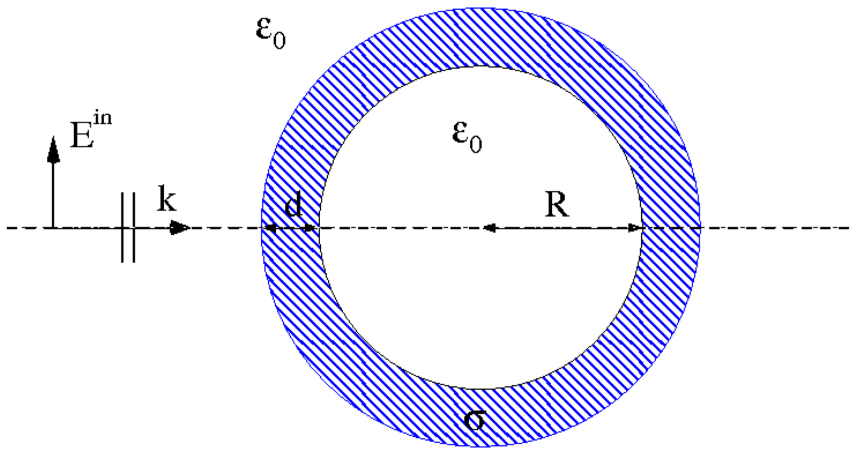
Frequency (from 100 MHz to 2.5 GHz)



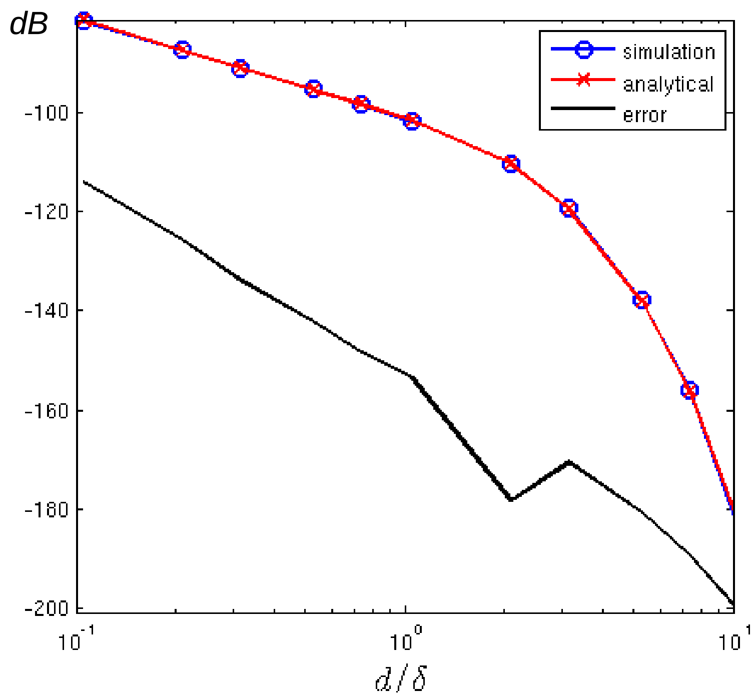
@ 250MHz

- 3.2 GByte
- 16 processors
- 526 s setup time
- 251 iterations
- 0.5 s per iteration

- spherical copper shell of thickness d and inner radius R=1m
- frequency: 47.7 MHz (k = 1, $\lambda = 1/2\pi$)
- skin depth $\delta = 9.46\mu m$

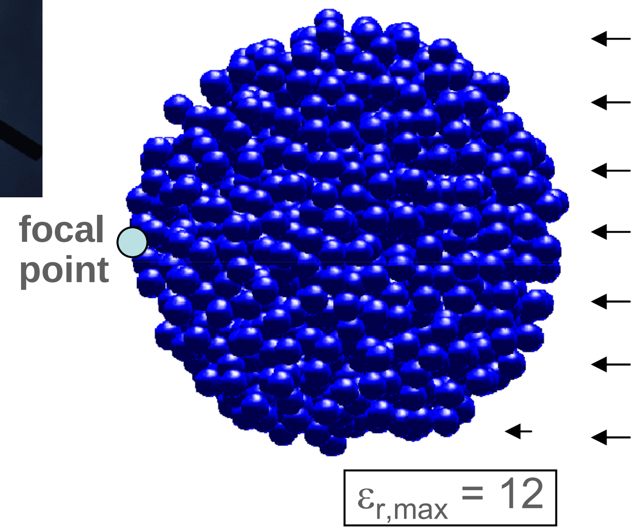


field penetration

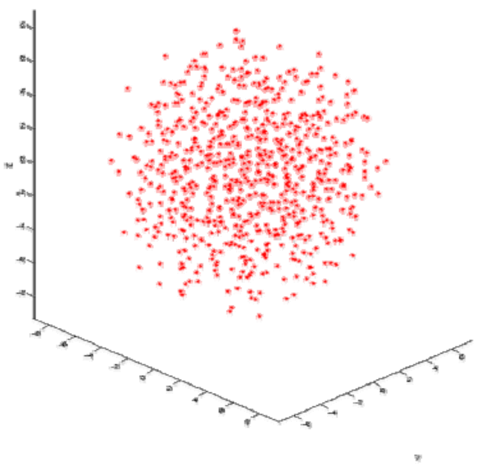


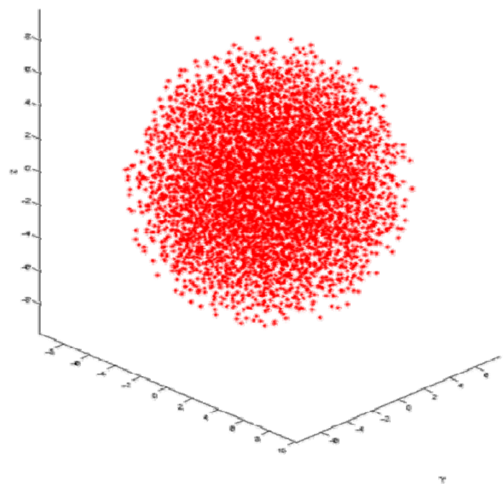
- **Lüneburg lens (radius = 8λ =80 cm)**
  - + inhomogenious refractive index
  - + $\varepsilon_r = (2 - R/R_{sphere})\ \varepsilon_{r,max}$

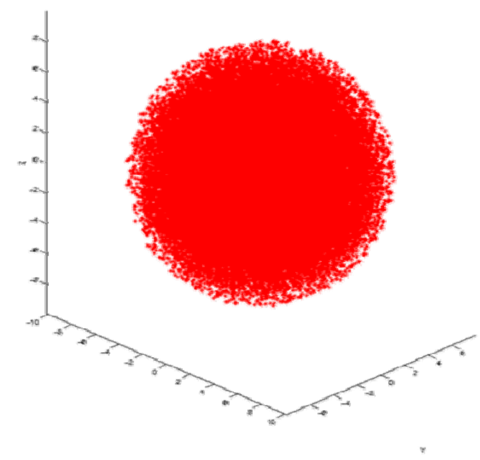
  modelled by identical spheres (ε_r = 12) but denser near the centre



- **3 test geometries**
  - R_{ss} = 2.4cm (669 small spheres)
  - R_{ss} = 1.2cm (5362 small spheres)
  - R_{ss} = 0.6cm (42899 small spheres)

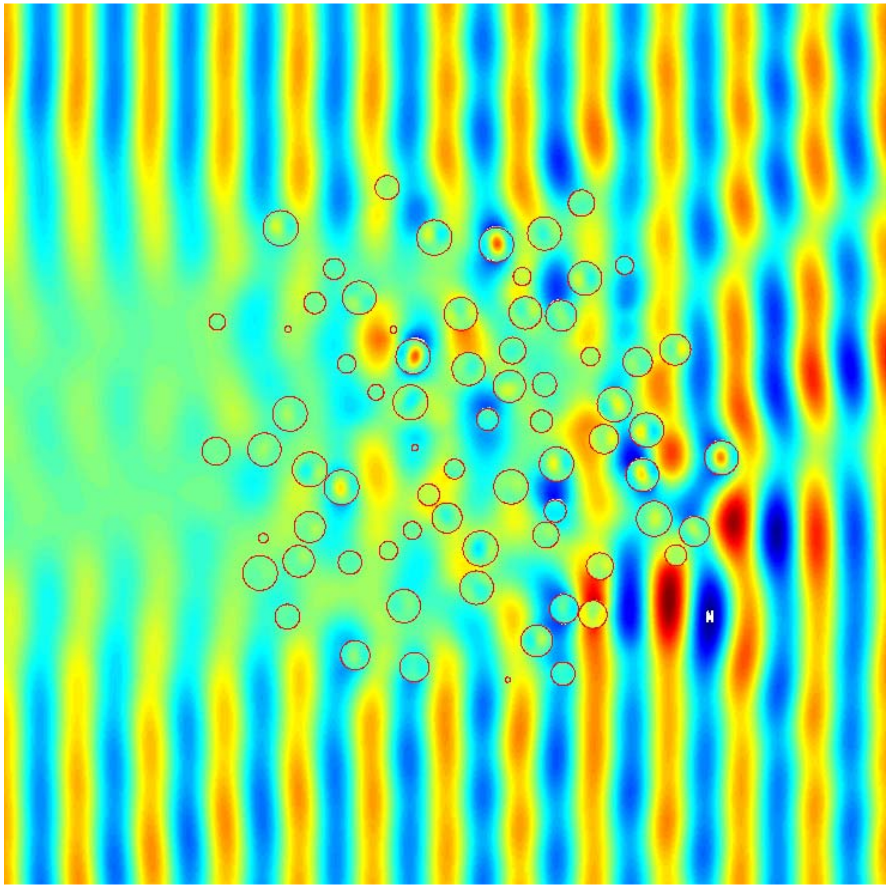






Lüneburg lens

- **669 spheres, 2 007 currents**

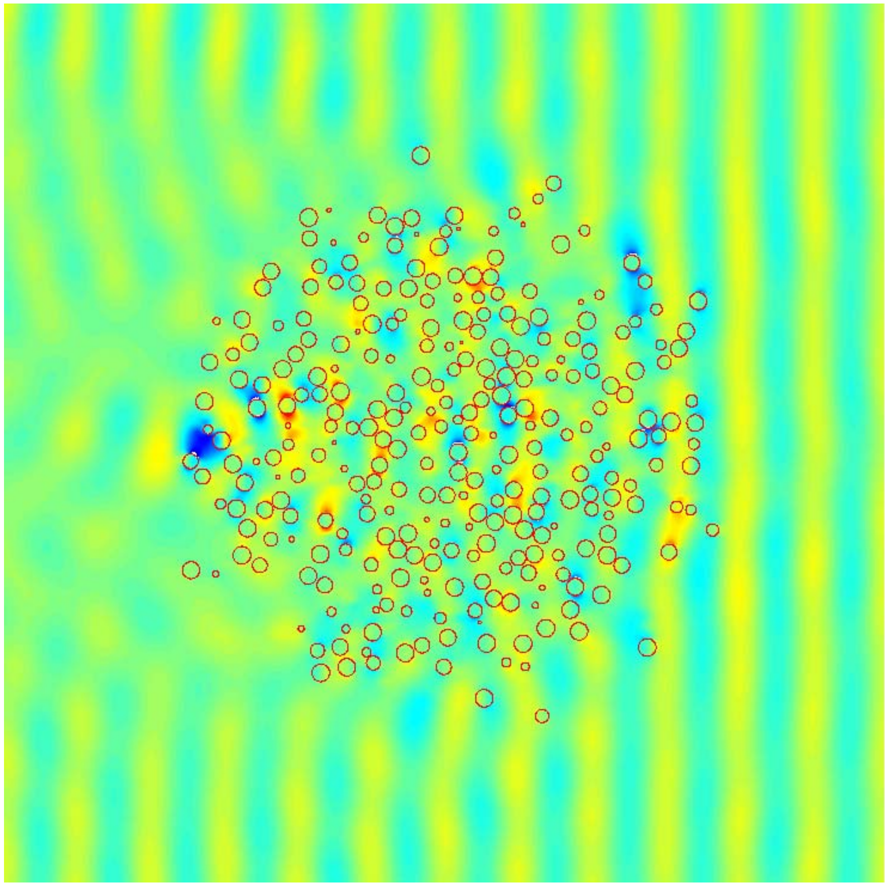


note:

+ the small spheres are indentical but their centre is not necessarily in the plane of the cross-section
+ each small sphere is modelled by a single equivalent electric current i.e. 3 scalar unknowns
+ all the interactions between the spheres are taken into account

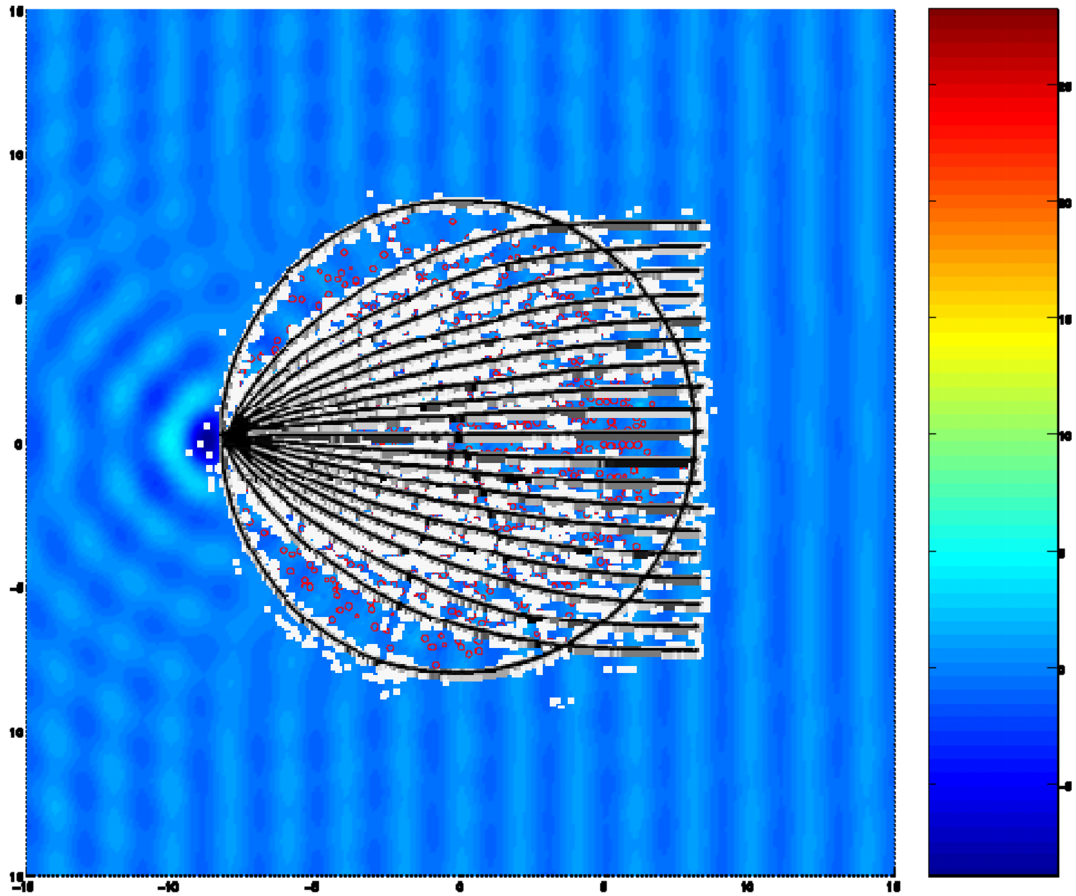
Lüneburg lens – cont.

- **5362 spheres, 16 086 currents**

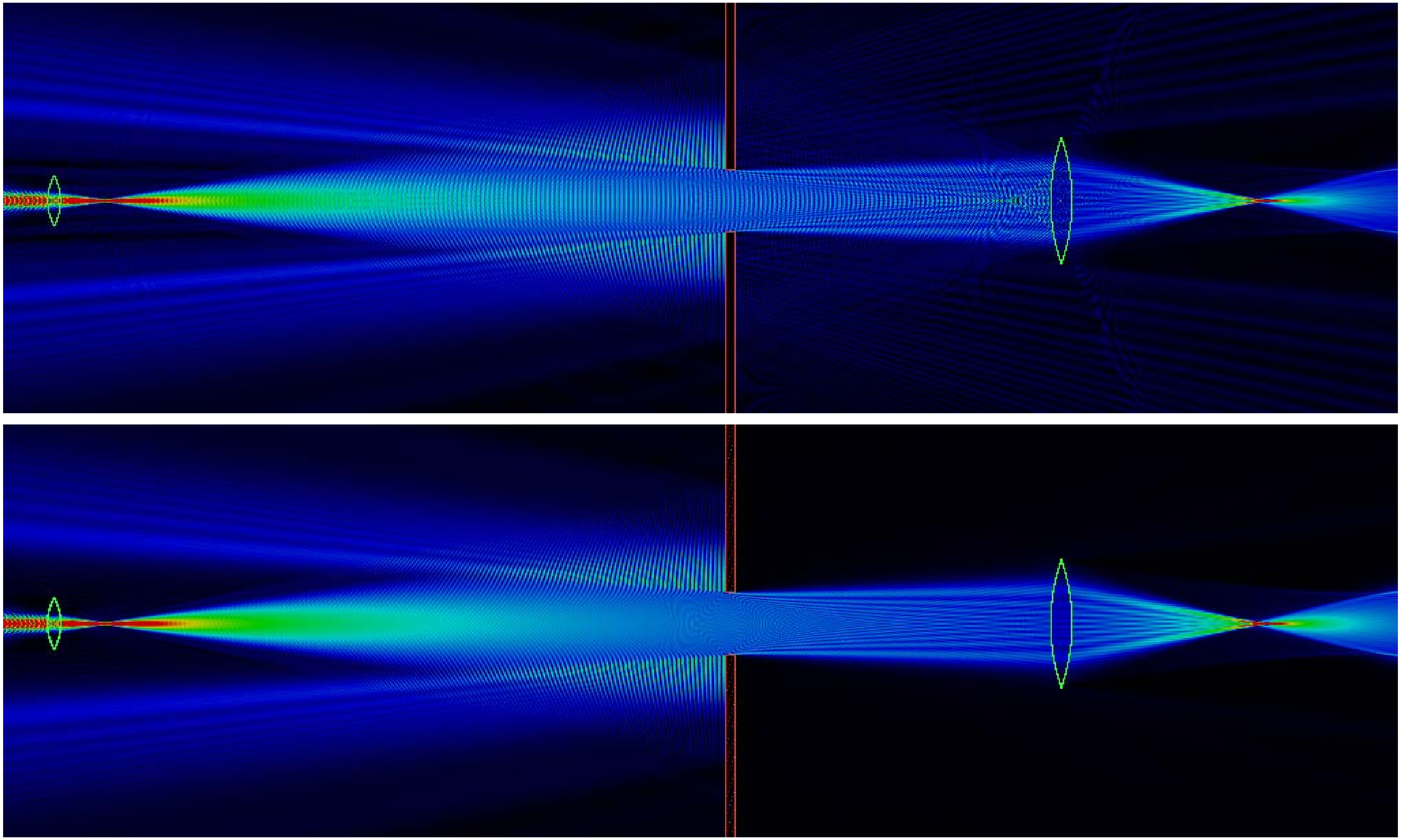


Lüneburg lens – cont.

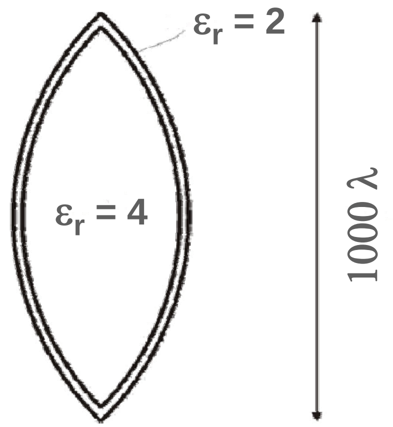
- 42899 spheres, 128 697 currents

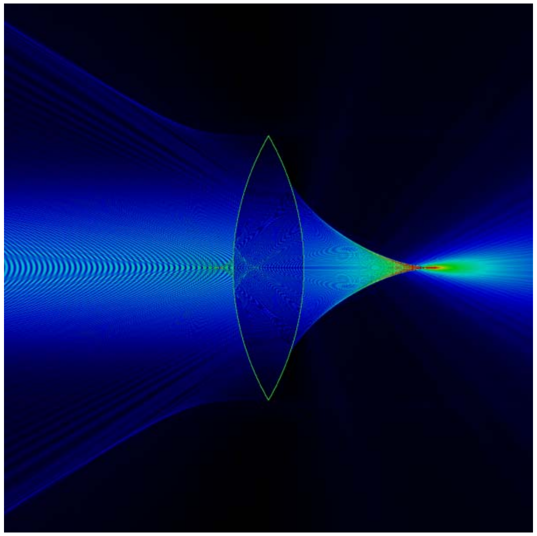


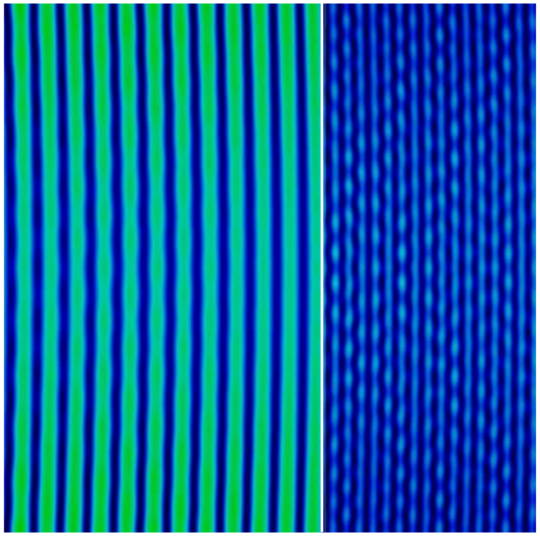
Lens system

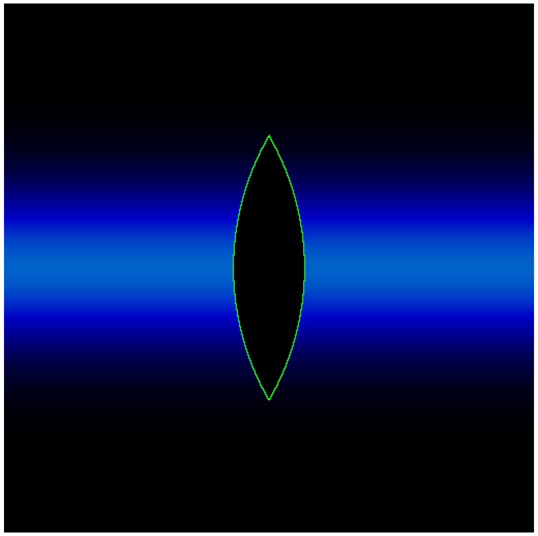


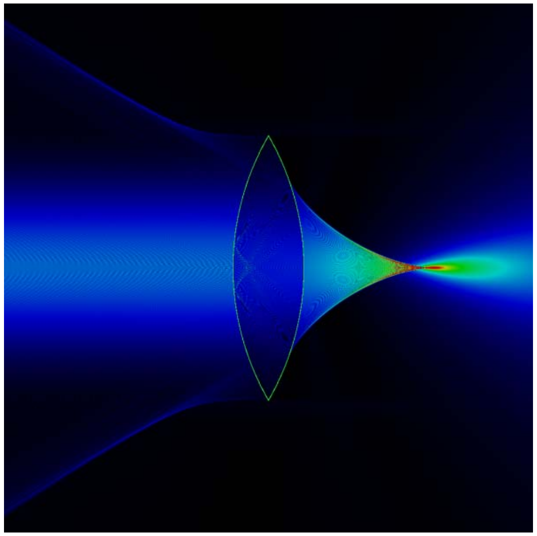
Coated lens

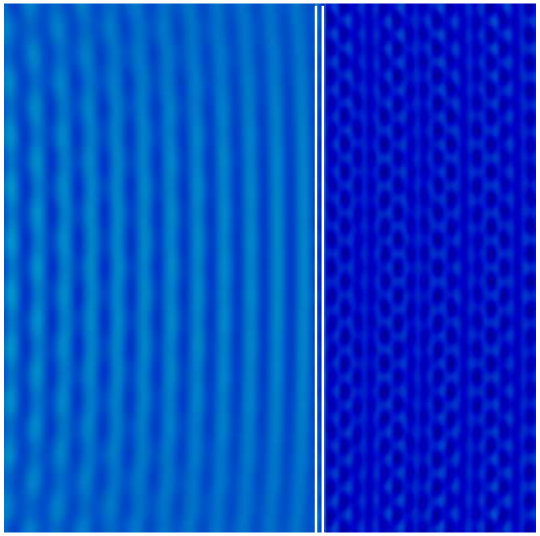






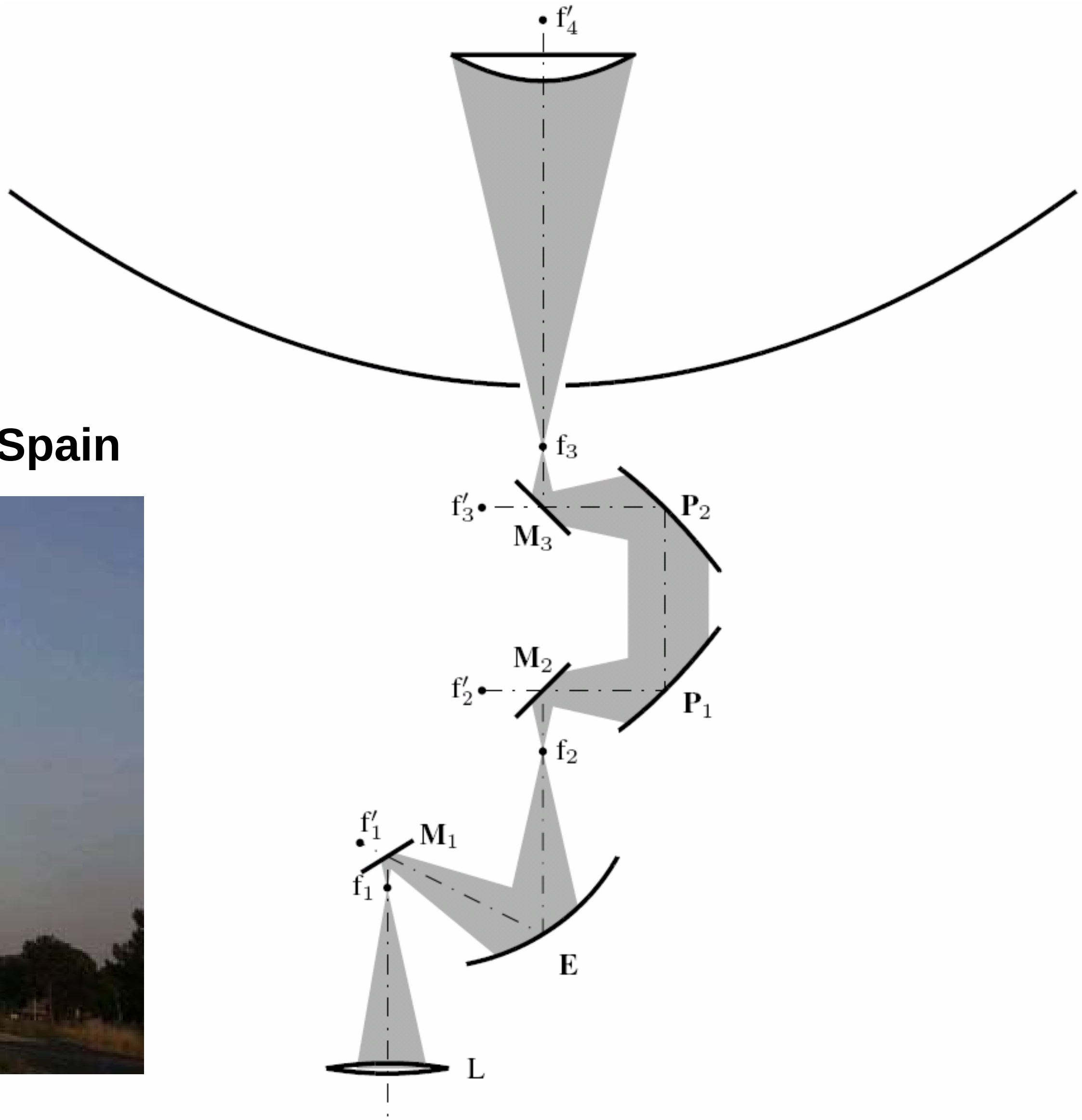




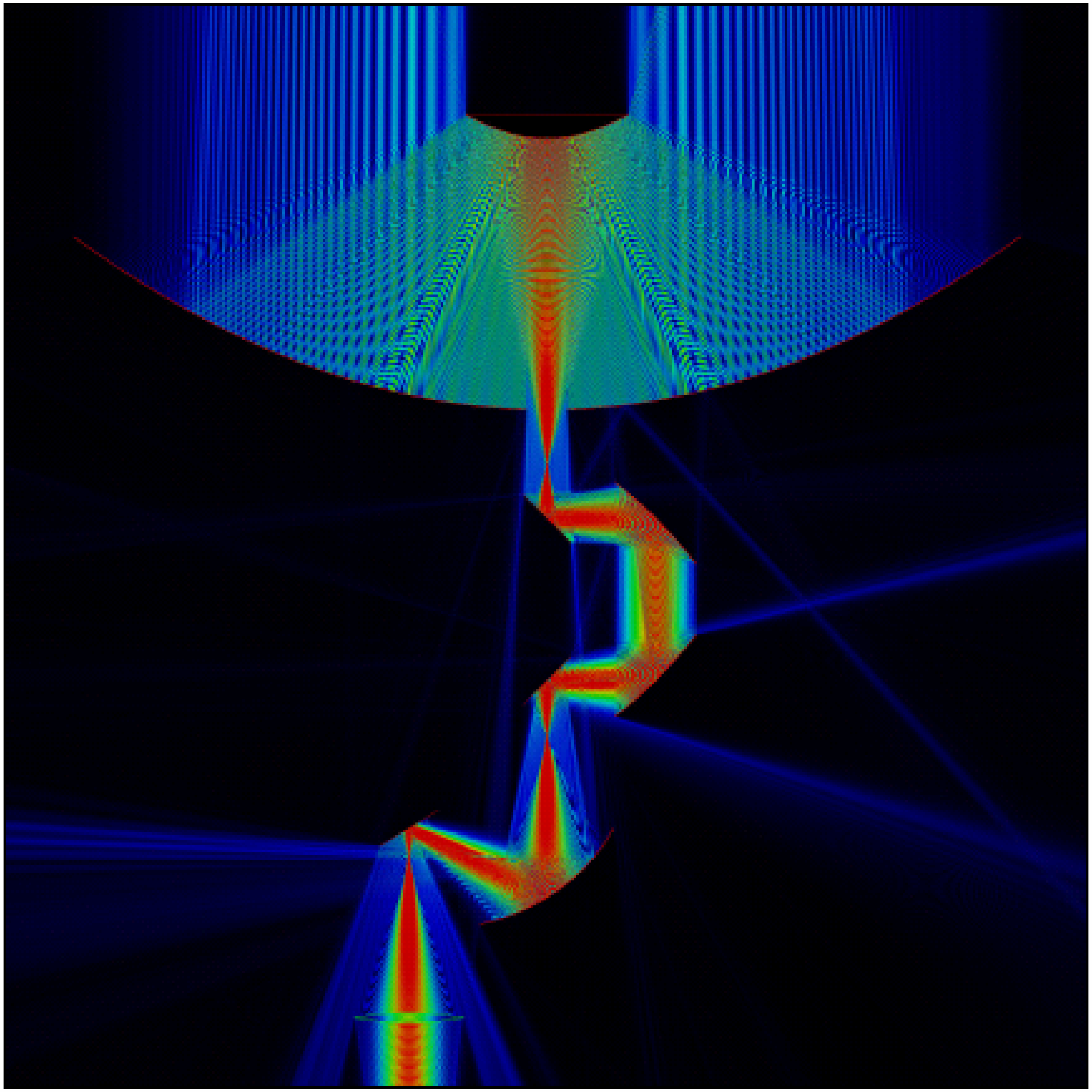


32 GHz
4000 λ in diametre
250 000 unknowns
7 minutes total solve time
16 processors

ESA DSA2, 35m Cerebros, Spain



Cassegrain Antenna – cont.



Future challenges

- hybridization with Finite Elements
- complex interconnect problems in *layered media*
- powerful time-domain FMM
- combination with ray theories
- ....

Thank you for your attention!

Questions?